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Allocation of Common-Pool Resources in an Unmonitored Open System

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***Abstract*—In an open system, system resources are managed by the contributors themselves. Because the participation needs to be voluntary and contributors true utility will remain unmonitored, proper communication among the participants is essential. In the discussed common-pool resource (CPR) problem, all the members need not be contributors, but the non-excludable component of the resource is required to be multiplied with each of the member's rivalrous component, and all these products are needed to be summed up to calculate the overall resource requirement. This characteristic applies to a typical power system optimization problem, where, if a customer group installs a common dynamic voltage restorer, voltage improvement can be treated as non-excludable quantity, while the peak load of individual customers can be treated as rivalrous quantity. In this work, we consider, the participants sharing the CPR contribute to form an open system to capitalize on 'economy of scale', while discouraging the unilateral free-riding benefit. Considering the benefit and average production cost curve represented by piecewise linear functions we have shown that the utility function is convex. Furthermore, for the given problem, we have numerically calculated the utility distribution scheme by solving an optimization problem.**

***Index Terms*—Common pool resources, voltage sag mitigation, carbon capture and storage, game theory, convexity, optimization.**

I. Introduction

In a socio-economic analysis, we usually assume that players or participants are rational [1], and rationality is the common knowledge for all the players [2]. A rational player may seek to procure goods from the other participants at a minimum cost, or sell it at the maximum available price, thereby inducing the allocation of goods to be 'efficient' [3]. However, asymmetry in the available information can bias players' decision-making ability [4].

Although both in common pool resources (CPR) and public goods, the players share natural or human-made resources, where, it is difficult to prevent any payer from the consumption of resources, the distinction among both kinds of resources is established [5], [6] based on the concept of rivalrousness [7]. Because of its rivalrousness nature, negligence in the monitoring of CPR goods will lead to an uncoordinated utilization of resources among the selfish and myopic players [8], [9], leading players into the 'tragedy of the commons' [10]. However, limited ability to comprehend this agelong social problem limits the ability to provide a detailed guidance [11]. Based on experimental evidence, it has been found that there is no unique way to solve the problem of the commons [12], [13], while the establishment of 'institution' creates consumption right in the CPR good, and incentivize the players in avoiding overuse [13]–[16]. Costly monitoring and sanctioning strategies are also needed to be in place [13], [14] to eradicate the opportunistic free-riding behavior of the selfish players [17]. Thusly, Pitt *et al.* [18] have discussed a community management system based on Ostrom's theories on social capital [19], where justice acts as social capital and is responsible for the successful collective action in socio-technical systems. However, the players can also share common social characteristics [20], and hence genuine trustworthiness can also achieve collective action compliance [21]. Individual players also can align themselves for sustainable extraction of resources while enabling internal monitoring and communication [22]. Furthermore, communication among contributors can result in an efficient strategy, which in turn will improve the quality of the available CPR resource [23], [24].

Existing contribution group formation strategies for the allocation of CPR goods are primarily based on the formulation developed by Walker *et al.* [25], which states that each player should receive a characteristic value or a utility in direct proportion to its contribution. Even if a non-cooperative behavior among the players materializes, the rational decision maker paradigm alleviates such an inferior outcome [26]. Besides, this inferior outcome solely relies upon the individual rationality of risk-averse myopic players, because individual players do not always follow the Nash solution concept [15], [27]. Although non-cooperative institutions may not always result in a tragic outcome, to improve the model outcome, treatment of the CPR resource allocation problem under cooperative strategy has

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been widely used. However, when the players have no information over trust and belief, individual players behave as payoff maximizers [28]. The solution can be substantially improved if the players design their own rules utilizing communication and coordination [29], [30]. Under the cooperative strategy, group rationality of the contributors generates sustainable benefit to all the contributors.

In the cooperative game theory, the challenge is to efficiently distribute the benefit arising out of cooperation among the contributors. One of the efficient distributions of the resources is based on individual rationality, group rationality, and Pareto efficiency, which is also known as the core [31] of a cooperative game. The core (if exists) includes infinitely possible outcomes while limiting the possible utility each player can achieve and suggests that no player can benefit from the successful coalition deviation. The core is stable if the characteristic function of the game is convex [32]. Therefore, the convexity of the core establishes strong incentives for cooperation. Driessen & Meinhardt [29] studies convexity and average convexity properties of transferable utility CPR (TUCPR) games. In addition to the core solution concept, Shapley value can also be used in the distribution of resources equitably and efficiently among the players [33]. Moreover, the Shapley value lies at the center of gravity of the core of a convex game. These properties hold for the transferable utility cooperative game without side payments [34]–[36].

In an open system, unlike natural resources, the provision of a central controller is unlikely, and the players themselves are simultaneous providers, allocators, and consumers [17], [18]. If the provision of resources are common to all the players, like traditional CPR, individuals face coordination dilemma for sustainable allocation. Terms such as 'justice,' and 'trustworthiness' may not be suitable if the considered strategy is of single-shot. In this article, a new kind of common-pool resources (CPR) has been discussed where players' resource requirements are the product of their rivalrous and the non-excludable component. If, the group members contribute to form an open system, since participation in this arrangement is completely voluntary, the system components may or may not provide information about actual benefit function of the required resource, and, without full disclosure, because, utility distribution is based on individual benefit function, there may be a significant benefit of 'defection' from the 'common faith,' leading to the free-riding of some potential contributors. However, even if the game is of a single shot, under the right utility distribution, the willingness of cooperation within the group can exist. One of the reasons behind cooperation can be the existence of an 'economy of scale'.

Remark 1: If the declining average production cost in a manufacturing process holds, then, there exists a provision of cooperation within the group.

Proof: Let, C_u be the unit average cost of production, and x_i is the individual demand of resource. If $C_u(\sum_i x_i) \leq C_u(x_i)$, then $\sum_i x_i C_u(x_i) \geq \sum_i x_i C_u(\sum_i x_i)$. Therefore, while the overall benefit from the common resource provision to the individual contributor remaining the same, player's contribution would be actually lower compared to the independent resource cost. ■

Even if a strong reason behind cooperation exists, we consider that mutual trust among the players can only be ensured by an appropriate payoff distribution. Although the distribution of utility according to the core can lead to individual and group rationality and Pareto efficiency, because of the non-excludable nature of the CPR resources, the benefit of free-riding still exists. However, in addition to distributing the generated utility completely among the contributors, the non-existence of the unilateral deviational utility will induce an internally stable allocation for the contribution group. Such a core concept is described in the literature as the free-riding proof core in a public good economy [37]. The utility distribution according to the free-riding proof core may lead the myopic players to gain the "right incentive" [11] for the group's sustainability.

To establish the existence of the free-riding proof core, we first show that the total utility generated by the group is convex, symbolizing the core solution concept exists. For simplicity, we have considered both marginal benefit function of the participants and the average cost function to be piecewise linear. Marginal benefit function is strictly decreasing and the average cost function is also non-increasing. We have also assumed that the open system CPR solution is available from a single manufacturer. Given a contribution group, because the rivalrous component is already given, we intend to find the optimal non-excludable component of the CPR good to be provided to the participants. Therefore, we can suitably convert our CPR good into a public good game and use free-riding proof core solution concept for solving this open-system CPR allocation problem with voluntary participation. Nevertheless, the proposed methodology is also shown to be applicable to the distribution of utility among a group of customers installing a dynamic voltage restorer (DVR) as a common voltage sag mitigation solution, and the installation of common carbon capture and storage solution.

II. Economics of CPR

Described following is an N-person single-stage game, where once within the group, each of the player ($i \in N$) is required to be provided with the CPR good to satisfy their demand. Let the CPR demanded by player i can simply be given by the product of two uncorrelated parameters say, α (≥ 0) and β_i (≥ 0). α is unknown but common to all participants and thereby can be termed as a non-excludable parameter. On the other hand, each player defines their requirement concerning β_i, and the whole group is to be provided with $\beta = \sum_{i \in N} \beta_i$. Therefore, β_i can be considered to be rivalrous or consumable. Overall consumption of CPR for each player $i \in N$ will be $\alpha\beta_i$. Once player i obtains $\alpha\beta_i$ of the overall provision $\alpha \sum_{i \in N} \beta_i$, the player's consumption does not affect the consumption of other players ($j \in N \backslash \{i\}$) within the group.

We have considered simultaneous protocol [38] in the proposed context, where each player bids privately. Each player internally calculates its marginal benefit by the willingness to pay function, which is represented as a piecewise linear function, and discloses its maximum marginal benefit from the CPR provision, which is denoted by a_i (≥ 0). Theoretical maximum non-excludable CPR provision is given by α_{MAX} (≥ 0) and is common and known to all the players. The mechanism for reaching to common α_{MAX} is not discussed in this paper. The pay-off scheme and the rivalrous consumption β_i is also common knowledge for all the players. The players can openly discuss their strategy before privately bidding their contribution strategy. Marginal willingness to pay $m_i^w(\alpha)$ for player i is given by,

$$m_i^w(\alpha) = \begin{cases} a_i\left(1 - \frac{\alpha}{\alpha_{MAX}}\right) & \text{if } 0 \leq \alpha \leq \alpha_{MAX} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Suppose the sub-set $S\,(\subseteq N)$ of the players forms a contribution group, by indicating true willingness to pay. Then, sub-set $N \backslash S$ is the set of free-riders (active or passive), with strictly zero willingness to pay. The aggregated marginal cost (m_S^w) based on the willingness to pay for the contributor set $S\,(\subseteq N)$ with respect to the non-excludable CPR provided (or simply, CPR provided), α can be given by,

$$m_S^w(\alpha) = \sum_{i\in S} m_i^w = \sum_{i\in S} a_i\left(1 - \frac{\alpha}{\alpha_{MAX}}\right) \tag{2}$$

We consider a single manufacturer case, where the manufacturer also bid alongside the players for the production right. If the average cost of production is monotonically decreasing, the manufacturer can be a rightful candidate in the discussed CPR provisioning problem. Manufacturer bids, and, will be paid according to its average revenue curve. The average revenue from the non-excludable CPR provision of α is also represented by piecewise linear function, with a negative slope (economy of scale exists) and can be given by,

$$A^P(\alpha) = c\sum_{i\in N}\beta_i - \alpha d\left(\sum_{i\in N}\beta_i\right)^2 \quad \text{if } 0 \leq \alpha < \frac{c}{2d\sum_{i\in N}\beta_i} \tag{3}$$

where, c, d are ≥ 0 and $\alpha\sum_{i\in N}\beta_i$ is the cumulative production. The average revenue is not defined outside the said bound. Also, consider the case, $c \geq 2d\alpha_{MAX}\sum_{i\in N}\beta_i$, which symbolizes that the maximum non-excludable CPR provision is limited by non-excludable CPR resource available at zero marginal production cost. Furthermore, the condition in equation (3) dictates that the average revenue is positive and total revenue is increasing for the manufacturer to participate in the CPR provision. Since all the players (bidding truthfully or not) are benefiting from the CPR provision, the CPR provision must include consumption by all the players independent of their contribution status. And for the contributors, the manufacturer's average and marginal revenue curves will be the CPR group's average and marginal cost curve. For the maximum utility generation, the marginal utility must be equal to the marginal cost of manufacturing. And hence, the equilibrium non-excludable CPR provision into the market, while S is the set of contributors in the CPR provision, can be calculated as,

$$\alpha_{MKT}^S = \alpha_{MAX}\frac{\sum_{i\in S} a_i - c\sum_{i\in N}\beta_i}{\sum_{i\in S} a_i - 2d\alpha_{MAX}\left(\sum_{i\in N}\beta_i\right)^2}$$
$$\text{if, } \sum_{i\in S} a_i \neq 2d\alpha_{MAX}\left(\sum_{i\in N}\beta_i\right)^2 \tag{4}$$

If, $S = \varnothing$ (or, none of the players contribute), then $m_\varnothing^w(\alpha) = \sum_{i\in\varnothing} a_i\beta_i\left(1 - \frac{\alpha}{\alpha_{MAX}}\right)$ will be $= 0$, while $\alpha_{MKT}^\varnothing$ can be defined to be 0.

The price of the CPR provision or the average revenue generated by the manufacturer ($A^P(\alpha_{MKT}^S)$) can be given by,

$$A^P(\alpha_{MKT}^S)$$
$$= c\sum_{i\in N}\beta_i - d\alpha_{MAX}\frac{\sum_{i\in S} a_i - c\sum_{i\in N}\beta_i}{\sum_{i\in S} a_i - 2d\alpha_{MAX}\left(\sum_{i\in N}\beta_i\right)^2}\left(\sum_{i\in N}\beta_i\right)^2$$
$$\text{if, } \sum_{i\in S} a_i \neq 2d\alpha_{MAX}\left(\sum_{i\in N}\beta_i\right)^2, \text{ and } \alpha_{MKT}^S < \frac{c}{2d\sum_{i\in N}\beta_i} \tag{5}$$

III. On the Convexity of the Players' Net CPR Utility

To obtain the core, the total utility captured or the characteristic function of the game can be defined as follows.

Definition 1: Let α_{MKT}^S be the non-excludable part of the CPR good provided at the unit cost of $m_{MKT}(\alpha_{MKT}^S)$, by the contribution group $S \subseteq N$, where the contributors aggregate reservation cost is given by m_S^C. The characteristic function $\Gamma : 2^N \rightarrow \mathbb{R}_{\geq 0}$ can be defined as

$$\Gamma^S := \int_{\alpha=0}^{\alpha_{MKT}^S} m_S^w(\alpha)\, d\alpha - A^P(\alpha_{MKT}^S)\cdot\alpha_{MKT}^S$$
$$= \begin{cases} \frac{1}{2}\alpha_{MAX}\frac{\left(\sum_{i\in S} a_i - c\sum_{i\in N}\beta_i\right)^2}{\sum_{i\in S} a_i - 2d\alpha_{MAX}\left(\sum_{i\in N}\beta_i\right)^2} \\ \quad \text{if } \sum_{i\in S} a_i \geq c\sum_{i\in N}\beta_i \\ \quad\quad > 2d\alpha_{MAX}\left(\sum_{i\in N}\beta_i\right)^2 \\ 0 \quad \text{otherwise} \end{cases} \tag{6}$$

Note that $\sum_{i\in S} a_i > 2d\alpha_{MAX}\left(\sum_{i\in N}\beta_i\right)^2$ is sufficient condition for the non-negativeness of the characteristic value. Ensuring $\sum_{i\in S} a_i \neq 2d\alpha_{MAX}\left(\sum_{i\in N}\beta_i\right)^2$ is essential for finiteness of α_{MKT}^S, and $A^P(\alpha_{MKT}^S)$. Also, given the characteristic function is non-negative, α_{MKT}^S will be strictly positive, if, $\sum_{i\in S} a_i > c\sum_{i\in N}\beta_i$, otherwise we define α_{MKT}^S to be zero. Furthermore, because $\frac{c}{2d\sum_{i\in N}\beta_i}$ is manufacturer's maximum CPR provision (otherwise, the total cost is decreasing with increasing production), if $A^P(\alpha_{MKT}^S) > 0$, condition $\alpha_{MKT}^S < \frac{c}{2d\sum_{i\in N}\beta_i}$ will ensure $c\sum_{i\in N}\beta_i > 2d\alpha_{MAX}\left(\sum_{i\in N}\beta_i\right)^2$, condition of our considered case.

Along with set S, let T be the additional set of contributors who are willing to join the contribution group. Then we have, $\sum_{i\in S} a_i \leq \sum_{i\in S\cup T} a_i$. Also conditions $\sum_{i\in S} a_i >$

$2d\alpha_{MAX}\left(\sum_{i\in N}\beta_i\right)^2$, and $\sum_{i\in S}a_i \ge c\sum_{i\in N}\beta_i$ need to be satisfied for the contributors to procure the good. Next, we will analyse, how utility generated by the contribution group changes with an increasing set of contributors. Also is evident that $\sum_{i\in S\cup T}a_i \ge \sum_{i\in S}a_i > 2d\alpha_{MAX}\left(\sum_{i\in N}\beta_i\right)^2$.

With $c\sum_{i\in N}\beta_i \ge 2d\alpha_{MAX}\left(\sum_{i\in N}\beta_i\right)^2$,

$$\begin{aligned}
&1-\frac{c\sum_{i\in N}\beta_i - 2d\alpha_{MAX}\left(\sum_{i\in N}\beta_i\right)^2}{\sum_{i\in S}a_i - 2d\alpha_{MAX}\left(\sum_{i\in N}\beta_i\right)^2} \\
&\quad\le 1-\frac{c\sum_{i\in N}\beta_i - 2d\alpha_{MAX}\left(\sum_{i\in N}\beta_i\right)^2}{\sum_{i\in S\cup T}a_i - 2d\alpha_{MAX}\left(\sum_{i\in N}\beta_i\right)^2} \le 1 \\
&\quad\Rightarrow \alpha^{S}_{MKT} \le \alpha^{S\cup T}_{MKT} \le \alpha_{MAX} \qquad (7)
\end{aligned}$$

In addition,

$$\begin{aligned}
A^P\left(\alpha^S_{MAX}\right) &= c\sum_{i\in N}\beta_i - d\alpha^S_{MKT}\left(\sum_{i\in N}\beta_i\right)^2 \\
&\ge c\sum_{i\in N}\beta_i - d\alpha_{MAX}\left(\sum_{i\in N}\beta_i\right)^2 > 0 \\
&\text{(by definition)} \qquad (8)
\end{aligned}$$

CPR good provided in this scenario increases with increasing group size, while the average price is decreasing, but remain positive. If some typical condition results in the CPR good provision to be negative, we define it to be zero. Therefore, the exchange of goods will only happen subject to individual contribution as $c\sum_{i\in N}\beta_i > 0$. In this regard, $d = 0$ is a special case. Therefore, satisfying the condition $\sum_{i\in S}a_i > c\sum_{i\in N}\beta_i \ge 2d\alpha_{MAX}\left(\sum_{i\in N}\beta_i\right)^2 \ge 0$, will ensure all of α^S_{MKT}, $A^P\left(\alpha^S_{MKT}\right)$, and $\Gamma(S)$ to be nonnegative.

Fact 1: Let $N = \{1, 2, \cdots \mathcal{N}\}$ be the set of all players participating in a contribution set of $\Omega = 2^N$. Then the measure induced by $\sum_{i\in S}a_i : \Omega \to \mathbb{R}_{\ge 0}$ is additive and non-decreasing.

Proof: Let $S, T \subseteq N$, then, because $a_i \ge 0, \forall i \in N$, $\sum_{i\in S\cup T}a_i = \sum_{i\in S}a_i + \sum_{i\in T}a_i - \sum_{i\in S\cap T}a_i$. ■

Lemma 1: Let $f : 2^\Omega \to \mathbb{R}_{\ge 0}$ be an additive, and non-decreasing function with $f(\varnothing) = 0$. Then if, $g : \mathbb{R}_{\ge 0} \to \mathbb{R}_{\ge 0}$ be a convex (concave) function with $g(0) = 0$ then $f \circ g$ is convex (concave) on Ω.

Proof: Since f is additive, $f(S\cup T) + f(S\cap T) = f(S) + f(T)$ (by definition). ■

Because f is nondecreasing, $f(S\cup T) \ge f(S), f(T) \ge f(S\cap T)$.

Therefore, $\exists\lambda \in [0, 1]$ such that,

$$\begin{aligned}
f(S) &= (1-\lambda)f(S\cup T) + \lambda f(S\cap T), \\
f(T) &= \lambda f(S\cup T) + (1-\lambda)f(S\cap T) \\
&\text{(From additivity property)}
\end{aligned}$$

Given g is convex (concave) with $g(0) = 0$, then, $\exists\delta \in (0, 1]$, $g(\delta x) \le (\ge)\delta g(x)$; while, $g(\delta^{-1}\delta x) \le (\ge)\delta^{-1}g(\delta x) \Rightarrow \delta g(x) \le (\ge)g(\delta x)$. And therefore, $g(\delta x) = \delta g(x)$.

Therefore, g is the first degree homogeneous function. Again, given g is also convex (concave),

$$\begin{aligned}
g(f(S)) &= g((1-\lambda)f(S\cup T) + \lambda f(S\cap T)) \\
&\le (\ge)(1-\lambda)g(f(S\cup T)) + \lambda g(f(S\cap T)) \\
g(f(T)) &= g(\lambda f(S\cup T) + (1-\lambda)f(S\cap T)) \\
&\le (\ge)\lambda g(f(S\cup T)) + (1-\lambda)g(f(S\cap T))
\end{aligned}$$

Adding,

$$g(f(S)) + g(f(T)) \le (\ge)g(f(S\cup T)) + g(f(S\cap T))$$

This completes the proof. ■

Fact 2: If f is a twice-differentiable function on an open interval $\mathcal{I}$, then f is convex on $\mathcal{I}$, if and only if $f''(x) > 0,\ \forall x \in \mathcal{I}$.

Theorem 1: The characteristic function game on (N, Γ) is a convex game.

Proof: Γ can be decomposed into a convolution of two nonnegative functions, f and g respectively, such that $\Gamma = f \circ g$; where, $g(S) = \sum_{i\in S}a_i$, and $f(x) = k_1(x-p)^2/(x-q)$, where, $k_1 = \frac{\alpha_{MAX}}{2}$, $p = c\sum_{i\in N}\beta_i$, $q = 2d\alpha_{MAX}\left(\sum_{i\in N}\beta_i\right)^2$ are ≥ 0, and $p \ge q$.

Then, $f'(x) = \begin{cases} \frac{(x-p)(x-(2q-p))}{(x-q)^2} & x \in [p, \infty) \\ 0 & \text{otherwise} \end{cases}$

Given that, $p \ge q$, we get, $2q - p \le p$ and therefore, $f'(x) \ge 0$ for $x \in \mathbb{R}$,

Again, define, $f''(x) = \begin{cases} \frac{2(q-p)^2}{(x-q)^3} & x \in (p, \infty) \\ 0 & \text{otherwise} \end{cases}$, which signifies $f''(x) \ge 0$, for $x \in \mathbb{R}$. Therefore, $f(x)$ is convex, and, using Lemma 1, the characteristic function game is also convex. ■

Fig. 1 depicts a pictorial representation to explain CPR provision.

Remark 2: Increase/decrease in α_{MAX} will increase/decrease non-excludable CPR provision, and the net utility generated by the players while decreasing/increasing its price.

This can easily be visualized independently from Fig. 1 and equation (3). It is also notable that both non-excludable CPR provision and the net utility generation are concave, and they saturate with increasing α_{MAX}. The price of such a provision is monotonically decreasing.

Remark 3: The coalition deviation by each contributor is not dominated by the true maximum contribution level.

Proof: Let, $A = \sum_{i\in S}a_i$ be calculated based on a truthful maximum contribution of each contributor. Now, if there exists a player i who wishes to be benefited by bidding its maximum marginal CPR provision in such a way that the total of the maximum contribution is determined to be A' that is greater than A, it can be observed that the true marginal benefit of such contribution is lower than the marginal cost of such provision.

The cost of such a provision can be provided by the deviant itself, or the whole group. But, sharing the cost by the deviant itself is inefficient. In contrast, if the cost of such provision is shared by the group, except the deviant, and the contributors except the deviant are aware of such benefit, then each of the deviants would like to falsify from their true maximum marginal

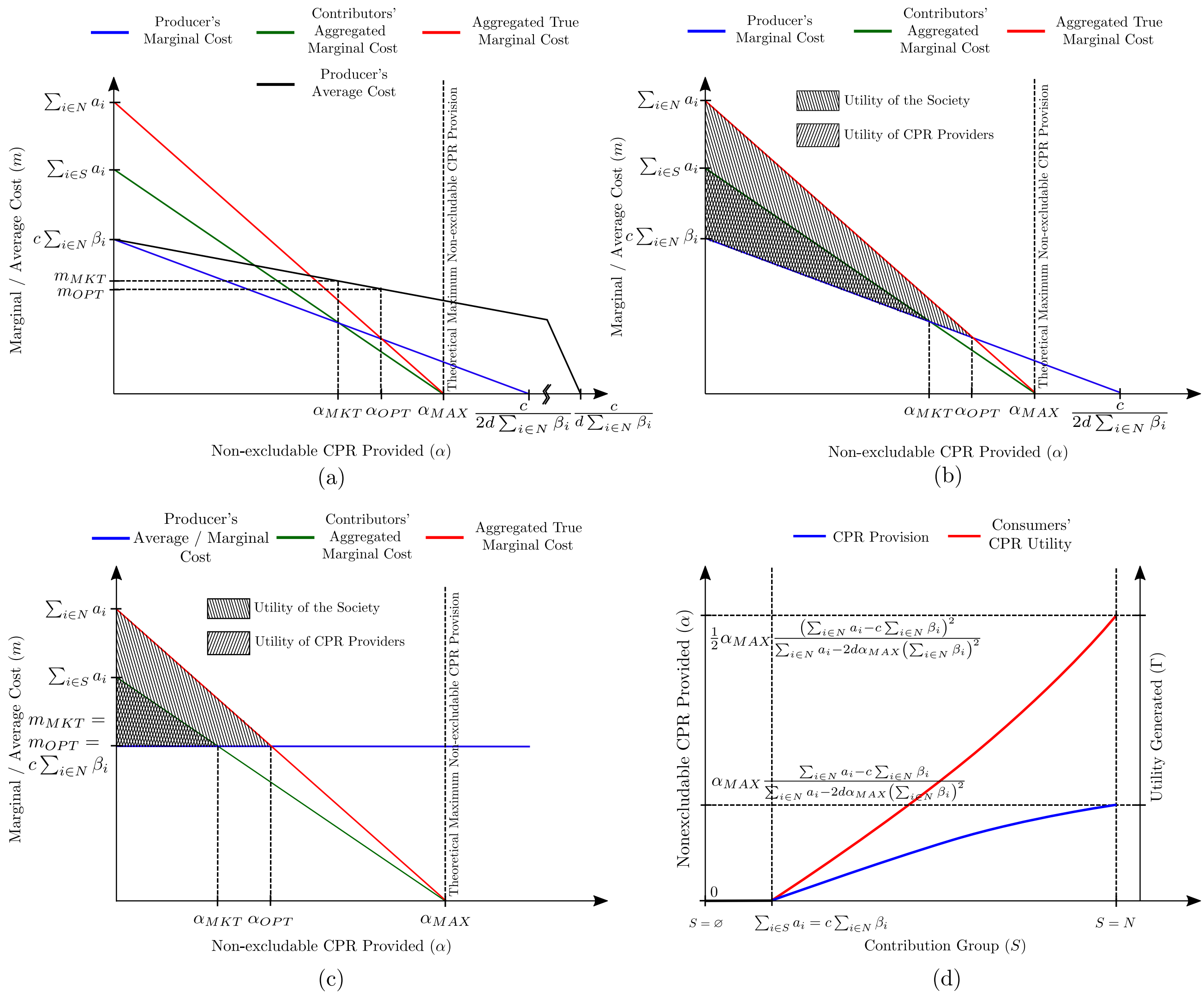

Fig. 1. Non-excludable CPR provision into the market for a successful contribution group formation. (a) Increasing the willingness to pay increases the non-excludable CPR provision. Manufacturer's average cost of production is decreasing, resulting in the price of non-excludable CPR provision to be also decreasing with increasing non-excludable CPR provision. α_{MAX} indicates the theoretical limit. (b) Decreasing average cost of CPR provision and increasing willingness to pay of each of the appropriators increases the contributors' net utility with increasing appropriation, which is limited by the aggregated true social utility. Average production cost multiplied by non-excludable CPR provision will be equal to the area under the curve of the marginal production cost function. Thereby, total utility generated as presented in equation (6) can also be represented by the area lying between marginal aggregated benefit function and marginal cost function. m_{MKT} becomes the price of the CPR good when a contribution group $S(\subset N)$ forms, providing non-excludable CPR of α_{MKT} among all the participants. Participation of all the players (N) would result into the utility to be true social utility, available at the price of m_{OPT} with non-excludable CPR provision of α_{OPT} available among all the participants. Similarly, (c) shows that if the average cost of CPR provision remains constant, the CPR provision and players' net utility is still increasing with increasing contribution level. Constant average production cost indicates constant marginal production cost, and therefore, total utility generated to be lying between marginal aggregated benefit function and average cost function. And, (d) shows the convex characteristic of players' overall utility function and the concave nature of the group's non-excludable CPR provision. Also note that, if the contributors aggregated maximum marginal benefit from non-excludable CPR provision is less than the average cost of zero non-excludable CPR provision, then, both non-excludable CPR provided and the utility generated are defined to be zero.

contribution level while expecting others to pay for it, leading to a contradiction.

However, in the condition where A' is less than A, the utility, and the marginal cost of the contributor presented to the group are lower than the true value. Although the resource provided to the group is not efficient at the quantity of non-excludable CPR provided, the group's marginal utility is still higher compared to the marginal cost of production. And therefore, deviants cannot be barred from gaining within the group from free-riding. ■

IV. The Optimal Contribution Group

Let, u_i^S be the utility received by the participant i, while S be the set of contributors. To obtain the optimal set of contributors, we need to maximize the utility derived by the contributors subject to the utility is being distributed according to the core, the distribution is unilateral coalition deviation proof and there does not exist any other group satisfying the core and free-riding proof property that is weakly dominated by our optimal contribution group. Therefore a utility distribution to be in the free-riding proof core, each contributor must receive more than its free-riding utility. Implying, the total utility generated by the group must be more than the cumulative utility to be distributed among the contributors to prohibit them from unilaterally deviating the contribution group.

The optimization problem (9)–(16) maximizes player's net utility following the free-ride proof core solution concept [37]. Although according to the core solution concept, infinitely many possible utility distributions are achievable, our objective is to obtain the coalition group generating the maximum utility and one of the potential utility distribution strategy. If the utility distribution u_i^S, and $u_i^{S'}$ under the contribution group of $S, S' \subseteq N$ satisfy free-ride proofness, then the utility received under any of the group is more than unilateral deviational utility (as shown in equation (10) and (13) below); Each of the utility distributions is in the Foley's core [39], (Equation (11), (12) and (14), (15)); and the utility distribution under the optimal contribution u_i^S is subgroup deviation proof (as shown in equation (16)).

$$\max_{S \in 2^N, U^S \in \mathbb{R}^N_{\geq 0}} \sum_{\forall i \in S} u_i^S \tag{9}$$

where,

$$u_i^{S \setminus \{i\}} \leq u_i^S, \quad \forall i \in S \tag{10}$$

$$\sum_{\forall i \in T} u_i^S \geq \Gamma^T, \quad \forall T \subseteq S \tag{11}$$

$$\sum_{\forall i \in S} u_i^S = \Gamma^S, \quad \exists S \in 2^N \tag{12}$$

such that,

$$u_j^{S' \setminus \{j\}} \leq u_j^{S'}, \quad \forall j \in S' \tag{13}$$

$$\sum_{\forall j \in T'} u_j^{S'} \geq \Gamma^{T'}, \quad \forall T' \subseteq S' \tag{14}$$

$$\sum_{\forall j \in S'} u_j^{S'} = \Gamma^{S'}, \quad \exists S' \in 2^N \setminus S \tag{15}$$

to not satisfy,

$$u_i^{S'} \geq u_i^S, \quad \forall i \in S' \text{ and } u_i^{S'} > u_i^S, \quad \exists i \in S' \tag{16}$$

Remark 4: If the contribution level of each player is chosen in such a way that $\sum_{j \in S \setminus \{i\}} a_j \leq 1.25c \sum_{j \in N} \beta_i$ are satisfied $\forall i \in S$ for a constant average cost of production, but $\sum_{j \in S} a_j$ is atleast $c \sum_{j \in N} \beta_i$, while, $d = 0$, then such contribution group will always be in an free-ride proof contribution group (utility generated is more than aggregated utility received by unilateral coalition deviation).

For $d \geq 0$, if $\sum_{j \in S \setminus \{i\}} a_j \leq c \sum_{j \in N} \beta_i$ is satisfied $\forall i \in S$, the contribution group will always be in an free-ride proof contribution group. The contribution group obtained after free-riding by one of the potential contributors will always result in further lowering of the utility received by the free-riders. Besides, in a free-riding proof core, there exists no coalition where players deviate to form another contribution group which will receive more utility, thus ensuring full stability against all coalitional blocking.

However, the utility generated to be more than the utility to be distributed to prohibit unilateral free-riding behavior does not signify the existence of the core. Therefore, to obtain the best fairing possible contribution groups satisfying free-ride proof core property, we have resorted to solving the optimization problem. The formulation of the optimization problem involves mixed integer non-linear programming (MINLP). The model is solved using the General Algebraic Modeling System (GAMS) utilizing the solver couenne. Although the optimization problem is solved for a hypothetical dataset to prove its applicability, it is evident that the proposed methodology can as well be applicable to a real-world problem with an actual dataset. The solution obtained from GAMS is further verified using Matlab. Furthermore, solution shown in Fig. 2 has been numerically computed by solving the optimization problem, which can also be analytically verified.

V. Examples

There exist several scenarios under which goods of the described characteristics may exist. Following two different scenarios are presented as examples:

Voltage Sag Mitigation: In an electricity network, the dynamic voltage restorer (DVR) is a series-connected device and has the voltage sag mitigation ability only to a set of electricity customers connected to its downstream [40]. Because the voltage sag performance improvement or the injected voltage with the use of DVR is independent of players' contribution status, the utility so generated will be non-excludable; but, given that the loading or the current rating of the DVR will be an algebraic sum of individual loading, the DVR requirement will be rivalrous as well. Considering the willingness to pay for the voltage sag improvement solution is a piecewise linear function of allowable minimum residual voltage and is independent of the duration, the problem formulation detailing the aggregated willingness to pay and the cost function are presented as demonstration 1.

Demonstration 1: Let, V_{max} be the theoretical maximum voltage that the DVR can inject, I_i be the peak loading of the customer i, where $i \in N$ and a_i be the maximum marginal willingness to pay for the customer i, where, $i \in S$, and S is the set of contributors ($S \subseteq N$). Let, the marginal willingness to pay, which can be obtained from the customer's cost of process failure, and is proportionately decreasing with increased voltage injection, V (≥ 0). Then, the aggregated marginal willingness to pay, $m_S^w(V)$, can be given by,

$$m_S^w(V) = \sum_{i \in S} a_i \left(1 - \frac{V}{V_{max}}\right) \tag{17}$$

Assuming the existence of prefabrication in the construction process, the economy of scale exists [40], [41] and if the decreasing production cost is allowed to be passed on to customers, they can choose to procure a common mitigation solution. The rating of the DVR can be given by, $V \sum_{i \in N} I_i$. Let, c and d be the parameters of the linearized monotonically decreasing cost function, the average production cost function, $A^P(V)$, can be

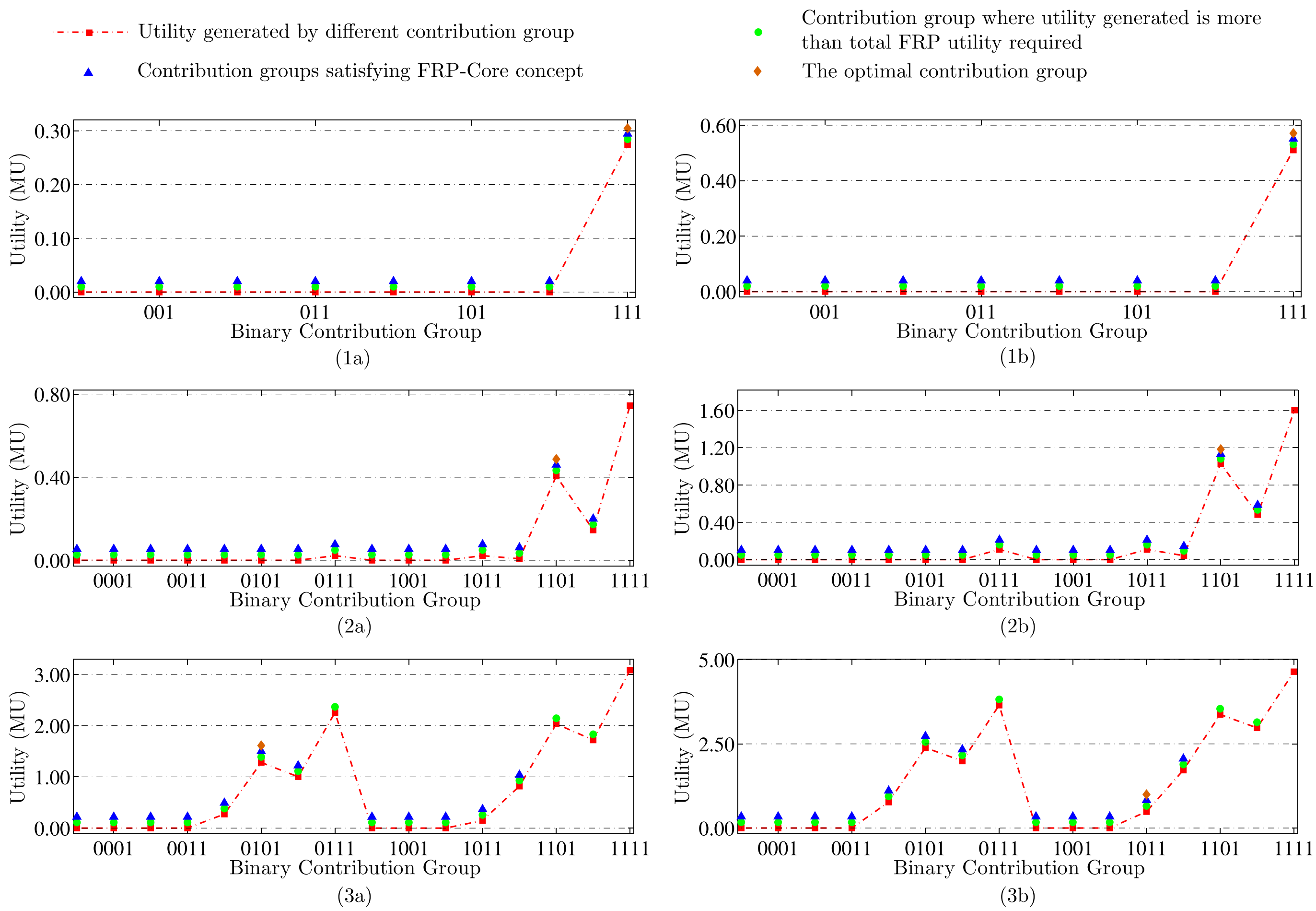

Fig. 2. Analysis of CPR provision into the market for different contribution levels. In the binary contribution group 0/1 signifies non-contribution/contribution status. The magnitude of c, β, and α_{MAX} in each of the possible group is 2MU/β^2, 1, and 0.8 respectively. MU refers to an arbitrary monitory unit. (1a) and (1b) Analysis for maximum contribution level of 2.50, 2.70, and 3.20 MU/$\alpha\beta$ respectively, with $d = 0$ MU/($\alpha^2\beta^2$) in (1a) and $d = 0.27$ MU/($\alpha^2\beta^2$) in (1b). Under this condition, each contributor likes to participate in the contribution group because the total utility generated by each possible unilateral deviation is zero. (2a) and (2b) Analysis for maximum contribution level of 4.20, 4.20, 1.50 and 3.00 MU/$\alpha\beta$ respectively, with $d = 0.00$ MU/($\alpha^2\beta^2$) in (2a) and $d = 0.27$ MU/($\alpha^2\beta^2$) in (2b). In (2a) contribution groups 0111 or 1011 or 1100 will be unilateral coalition deviation proof. But for contributor 4 (with maximum contribution level of 3.00 MU/$\alpha\beta$), $c\sum_{j\in N}\beta_i \le \sum_{j\in S\setminus\{i\}} a_j\beta_j \le 1.25c\sum_{j\in N}\beta_i$. Therefore, 1101 is also unilateral coalition deviation proof. Similar condition can also be reached for 1110. Furthermore, equations representing the core and unilateral deviation proofness shows all the groups, except 1111, satisfies free-ride proof concept. Among them, group 1101 generates maximum utility that is not blocked by other groups in free-ride proof contribution group. Similar conclusion can be made for (2b) as well. (3a) and (3b) Analysis for maximum contribution level of 2.50, 10.70, 3.20, and 4.21 MU/$\alpha\beta$ respectively, with $d = 0.00$ MU/($\alpha^2\beta^2$) in (3a) and $d = 0.27$ MU/($\alpha^2\beta^2$) in (3b). In (3a), group 1101, 1110, and 0111 does not satisfy free-ride proof contribution group concept. Group 0101 generates highest utility, belongs to free-ride proof core, and is not blocked by any other free-ride proof contribution group, and hence is an optimal solution. In (3b), among all free-ride proof contribution group, the group 0101 generating highest utility is unilaterally blocked by other groups, not satisfying free-ride proof core contribution property. Therefore, 1011 is selected to be an optimal contribution group.

given by,

$$A^P(V) = c\sum_{i\in N} I_i - dV\left(\sum_{i\in N} I_i\right)^2 \quad \text{if} \quad V < \frac{c}{2d\sum_{i\in N} I_i} \qquad (18)$$

Carbon Capture and its Storage (CCS): CCS employing the use of either advanced technologies or natural resources can significantly reduce the overall emission level from the use of fossil fuels. Carbon emission is not uniform across all regions. Besides, the risk associated with carbon emission is not localized, because, the consequence of increased emission or reduction in decreased utility may not be uniform across all the regions. For example, because of the rise in global mean sea level with an increased presence of carbon dioxide, low-lying coastal cities are in the comparatively higher level of threat of being submerged into the sea. Associated problem formulation highlighting the calculation of the aggregated willingness to pay and the cost function is shown as demonstration 2.

Demonstration 2: Let, N be the set of all regions, of which S be the set of countries willing to contribute in carbon capture and storage. For simplicity, it can be safely assumed that humans are sole excreter of carbon dioxide. Let, V_i be the amount of carbon dioxide excreted by a region i, where, $i \in N$. Let, a_i be the maximum willingness to pay for the set of contributing regions S, where, $S \subseteq N$. Also, the marginal benefit of the fractional absorption of carbon dioxide, f (≥ 0), can be assumed to be linearly decreasing, and the theoretical limit of the carbon dioxide reduction to be, f_{max}. Then, aggregated marginal willingness to pay, $m_S^w(f)$, can be given by,

$$m_S^w(f) = \sum_{i \in S} a_i \left(1 - \frac{f}{f_{max}}\right) \qquad (19)$$

Observe that the captured carbon to be stored is rivalrous and the reduction in carbon across all the regions is non-excludable. Furthermore, it is observed that there exists an economy of scale in carbon storage technologies (i.e., larger size storage plants incur lower per-unit investment and operating costs) [42]. The existence of the economy of scale may provide a strong motivation for the formation of the contribution group. The captured carbon to be stored will be, $f \sum_{\forall i \in N} V_i$. Let, c and d be the parameters of the linearized monotonically decreasing average cost function, the average cost of carbon dioxide storage, $A^P(f)$, can be given by,

$$A^P(f) = c \sum_{i \in N} V_i - df \left(\sum_{i \in N} V_i\right)^2 \quad \text{if} \quad f < \frac{c}{2d \sum_{i \in N} V_i} \qquad (20)$$

Comments: Similar to equation (2) and (3), equation (17) and (18) in demonstration 1, and equation (19) and (20) in demonstration 2 represents the aggregated willingness to pay and the cost function respectively. Linearized benefit and cost function ensures that the utility function is convex, while ensuring the applicability of the proposed solution methodology. Next, (4) and (6) can be used to obtain the optimal ΔV and f and optimal utility respectively for each of the contribution group S $(\in 2^N)$. Subsequently, the optimization problem outlined using (9)–(16) as discussed in Section IV can be employed for calculating both the optimal contribution group generating maximum utility and the utility distribution requirement for both demonstrations 1 and 2.

VI. Conclusion

The main purpose of the proposed research is to analyze one of the conditions under which players in an open system can voluntarily contribute to a CPR good provision. It is shown that if the average cost of production is constant or monotonically decreasing, or 'the economy of scale' exists in the manufacturing process, the appropriators are incentivized to invest in a common provision. Considering a piecewise linear benefit function, and a piecewise linear cost function, the common CPR provision to the contributor and the total utility are calculated for the single manufacturer case. It is shown that the utility of CPR provision is non-decreasing and convex, which is an essential condition for the existence of the core.

Since the participation in the contribution group is voluntary, the free-riding provision exists. To reduce the impact of free-riding, the utility generated by the contributors needs to be distributed according to the free-ride proof core solution concept. Although the free-riding behavior of some the contributors cannot be prevented, the utility received by the free-riders will always be less than the utility received if the free-rider choose not to unilaterally deviate.

The optimization model presented in this paper is not only limited to power system economics problem but can also be used in various engineering and social science applications.

As a part of future work, we wish to alleviate the considered case where maximum non-excludable CPR provision was limited by zero marginal procurement cost of the CPR resource. Contribution group formation if the players are weakly excludable, and are allowed to form multiple clusters is also a possible direction of our future work.

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