Preprint of the manuscript.
Full text available at: https://ieeexplore.ieee.org/document/9310210
Cite as: S. Majumder, S. A. Khaparde, A. P. Agalgaonkar, S. V. Kulkarni and S. Perera, "Graph Theory Based Voltage Sag Mitigation Cluster Formation Utilizing Dynamic Voltage Restorers in Radial Distribution Networks," IEEE Trans. Power Del., vol. 37, no. 1, pp. 18-28, Feb. 2022.

Graph Theory Based Voltage Sag Mitigation Cluster Formation Utilizing Dynamic Voltage Restorers in Radial Distribution Networks

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***Abstract*—Voltage sag mitigation utilizing dynamic voltage restorers (DVRs) can be classified as a common-pool resource (CPR) good. However, the ability of DVRs in improving voltage sag performance of only downstream customers provides the ability to exclude the non-contributors selectively. Therefore, unlike traditional CPR goods, the DVR allocation problem can give rise to partial excludability. Here, the non-collocated customers have been divided into feasible clusters through suitable positioning of DVRs using the proposed graph-partitioning principle. In the absence of trustworthiness, especially with the participation of electricity supply companies, the participants may ask an external agent and share their willingness-to-pay information to design the optimal set of clusters. Alternatively, the customers, including electricity supply companies, can also share internal information as an open system. Strategies for sharing internal information to avoid free-riding are also discussed. The utility distribution would ensure the viability of contribution group formation. Here, the results from three different utility-distributing solution concepts, such as an alternative definition of the core, the nucleous, and Shapley value, are compared. The simulation results are numerically verified for a small scale system and validated utilizing a large scale system. The proposed methodology can be used for any real-world system that follows similar properties.**

***Index Terms*—Clustering, combinatorial optimization, common-pool resources, cooperative game theory, dynamic voltage restorer, graph theory, optimal contribution group, voltage sag mitigation.**

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NOMENCLATURE

Sets

$\mathrm{V}_{\text{leaf}}, \mathrm{V}_R$	Set of leaf and root nodes respectively.
$\mathrm{V}_{ic}, \mathrm{V}_g$	Set of nodes forming clusters ic and g respectively.
$\mathscr{M}, \mathscr{M}_{(\cdot)}$	Sag mitigation device identifiers.
Ω	Power set constituting of all the participants in the common-pool resource game, including the null set.
G_{Ro}	Undirected rooted tree, represented by a set of vertices (V), and a set of edges (E).
G_{Su}	An undirected subtree.
ic, g	Indices of clusters formed within the distribution network.
N	The universal set indicating all the participants in the common-pool resource provision game.
q, r	Indices representing the nodes of the tree, or the player participating in common-pool resource game.
S, T	Proper or improper subsets of the set of all participants, including null set.

Parameters

α_{MAX}	Maximum non-excludable common-pool resource requirement by the participants.
β_q	Rivalrous component of the consumption by a player (q); representing maximum load current of the customer located at a given node of the distribution network.
$\ell_{q,r}$	Fault rate of the branch connected between nodes q and r.
$\mathcal{D}_q$	Degree of a node (q) of the rooted tree (representing the distribution network).
$\mathcal{L}$	Fault rate threshold decided by the customers.
$\mathcal{Y}_q^S$	The contribution status of the customer located at leaf node (q), within a contribution group (S), ($\in \{0,1\}$).
$\mathfrak{B}_{q,r}$	A matrix representing the existence of an edge between nodes q and r of the tree ($\in \{0,1\}$).
$\mathfrak{T}^{S,T}$	A matrix, indicating whether a contribution group (S) is a proper subset of another contribution group (T), ($\in \{0,1\}$).
a_q	Maximum marginal willingness-to-pay of a contributor (q), or, an appropriate cluster member.
c, d	Parameters of the average cost function of the mitigation device, such as, dynamic voltage restorer.

M — An arbitrarily large positive real number.

Variables

α — Non-excludable part of the common-pool resource provision (representing common voltage injected by the dynamic voltage restorer).

$\alpha_{MKT}^{(\cdot)}$ — Optimal non-excludable common-pool resource provision for a contribution group (or cluster).

$\chi_q^{g,S}$ — A matrix, representing whether a node (q) joins a cluster (g), in a given contribution group (*S*).

$\varGamma^{(\cdot)}$ — Total utility generated by a contribution group (or cluster), in MU.

$\varGamma^{g,S}$ — Utility generated by a cluster (g), in an appropriation group (*S*), in MU.

$\mathfrak{D}_q^{g,S}$ — Degree of a node (q) of the tree for a cluster (g) in a contribution set (*S*).

Phi$_q$ — The Shapley value received by each player (q).

A^S — A vector representing whether a contribution group (*S*) is an optimal solution ($\in \{0,1\}$).

$\mathscr{C}(N)$ — Core utility distribution strategy.

$\mathscr{O}(S)$ — Alternative definition of the core utility distribution strategy.

$U^{g,S}$ — Nullity of a cluster (g), in a group (*S*), ($\in \{0,1\}$).

u_q — Utility received by a node (q), in MU.

$W_q^{g,S}$ — Belongingness of a node (q), within a cluster, (g), under a contribution set, (*S*), ($\in \{0,1\}$).

I. Introduction

With ever-increasing number of sensitive and critical equipment connected into the traditional distribution network with a single point of failure, and the possible colossal cost of voltage sags [1]–[3], it is crucial to improve the sag performance of the system significantly. Faults, the major reason behind voltage sags [4], can emanate in transmission, sub-transmission, and distribution networks and propagate to the customers premises in accordance with the network topology and associated parameters. Furthermore, the majority of the faults occurring in the distribution network is temporary in nature [5]. Delay in fault-clearing due to requisite coordination among the protection devices results in its inevitability [6]–[8]. The stochasticity of the associated economic impact arises from both the stochasticity of the occurrence of temporary faults and the probabilistic nature of the failure of sag-sensitive microelectronic control devices [6], [9], [10].

To eliminate the impact of inherently stochastic voltage sags and given the expensiveness of mitigation devices, such devices' allocation problem has been extensively studied in the literature. Different techniques, such as those in [11], [12], have been introduced for the allocation (capacity and location) of the flexible alternating current transmission system (FACTS) based devices (also known as custom power devices in medium/low voltage applications [13]). In [11], the minimization of the weighted sum of the number of sags of different magnitude ranges is proposed. While in [12], a cost-benefit analysis for the sag mitigation devices allocation problem was proposed. Use of 'nested logic' model for the optimal selection of voltage sag mitigation devices has been considered in [14].

The use of individual process failure characteristics has been considered for the cost-benefit analysis of a typical industrial plant in [15]. Reference [16] focuses on finding a correlation among the incentive provision from mitigation devices and financial losses incurred by a typical industrial customer. An optimal mitigation solution provision based on aggregated power quality performance improvement requirement (sag performance improvement is incorporated as a part of the considered index) is introduced in [17]. For accounting disappointment-rejoicing psychological perceptions of sensitive customers, a new premium power investment strategy is proposed in [18]. Tabu search based optimal rating and location identification for voltage sag mitigation devices is discussed in [19].

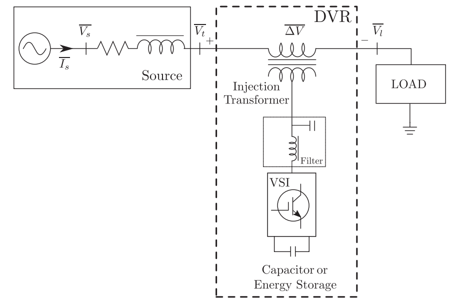

Fig. 1. Schematic diagram of a DVR.

Use of Dynamic Voltage Restorers (DVR) for voltage sag performance improvement [19], [20] is notable in the literature, and associated survey is available in [21]. Also, the consideration of multiple benefits from DVRs [22] is noteworthy. Some of these benefits have been considered in [23] as numerous objectives in a bi-objective optimization framework for a typical sensitive load. Placement of DVRs based on the maximum number of sag occurrence heuristic [24] and to minimize system average RMS variation frequency index - SARFI$_x$ [25] has been considered for reducing the impact of voltage sags. Optimal procurement of DVRs as voltage sag mitigation solution from a distribution system operator's perspective is discussed in [26]. However, the question of cost distribution among the customer still exists (ref. [18] have discussed this issue to a limited extent). It is to be noted that the uniform distribution of costs (for example, utility companies, here defined as electricity supply companies, install common sag mitigation solution, and charge every customer uniformly based on the maximum load demand), if multiple customers are being facilitated from common mitigation device, among all the customers independent of their willingness to pay will lead to a socially inefficient outcome. This is because, in this process, some of the sag insensitive customers will be forced to pay as well.

As shown in Fig. 1, a DVR can be realized by a voltage source inverter coupled with a DC link capacitor (with/without an energy storage device), an LC filter to eliminate switching harmonics, and an injection transformer connected in series with the distribution line. DVRs can intelligently inject the missing voltage with a certain phase angle, $\overline{\Delta V}$, in series with the source voltage, $\overline{V_s}$, to ensure the availability of a constant voltage at the load side, $\overline{V_t}$, during sag events. However, one of the downsides for the DVR installation is that it can only protect the downstream customers from reflected faults.

Since the sag-mitigation using DVRs is limited to the downstream loads [22], the boundary of the set of beneficiaries is

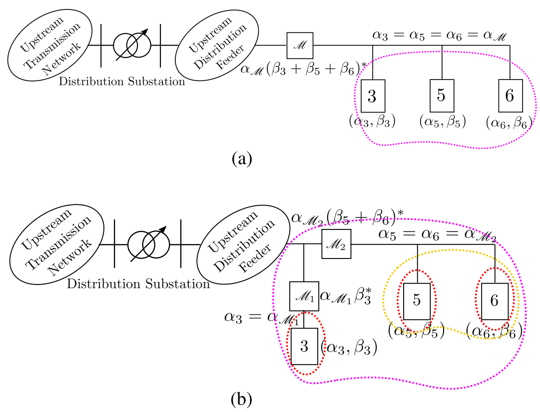

Fig. 2. Typical cluster formations with mitigation devices. (a) CPR formation group considering colocated customers. (b) Cluster formation with noncolocated customers.

strictly quantifiable. Reference [12] has shown the existence of an economy of scale while incurring costs for designing and installing DVRs, i.e., the average cost of a DVR is downward sloping. The existence of the economy of scale in the manufacturing process of the DVR induces a strong incentive for the formation of symbiotic groups for the common voltage sag mitigation solutions. It is impending that once the DVR sizing based on the maximum load demand of the downstream customers is decided, no-additional customer can further benefit from the common voltage sag mitigation solution provision. This results in the categorization of the common mitigation solution as a rivalrous good. Since the network's sagged condition within the contribution group remains equal and indiscriminate, the terminal voltage improvement through the injection of the voltage from DVR can be categorized as non-excludable. Furthermore as already discussed, the total resource requirement can be calculated by multiplying associated rivalrous and non-excludable components.

Since the mitigation solution is simultaneously non-excludable and rivalrous, the solution provisioning problem utilizing DVRs can be categorized as a common-pool resource (CPR) [27]. As indicated, once the DVR is designed, the associated benefit in terms of voltage sag mitigation solution provision will be enjoyed by all the group members (the set of all downstream customers) independent of their contribution status. If the economic utility (=benefit-cost) is not appropriately distributed, it will result in the existence of free-riders (those who enjoy the benefit of the resource without contributing). However, when the customers are partially excludable, their strategic removal is possible [28] if the inclusion of a free-rider is not beneficial for the group.

Therefore, in contrast to [27], where all the players are forced to be incorporated into the contribution group, as shown in Fig. 2(a), the possibility of the existence of multiple clusters, as shown in Fig. 2(b), is the primary focus of this paper. In this figure, $\mathscr{M}_{(\cdot)}$ is the voltage sag mitigation device, $\alpha_{(\cdot)}$ is the maximum possible voltage injection from the corresponding device, and $\beta_{(\cdot)}$ is the current rating of the loads. Because of their noncollocatedness in the distribution network, the customers can form multiple clusters based on their contribution level and relative location.

In Fig. 2(b), two feasible clusters, in red and yellow, have been identified and shown. Here, DVR $\mathscr{M}_1$ is incorporating customer '3' only, and DVR $\mathscr{M}_2$ contains customers '5' and '6'. Notably, only a specific set of clusters are viable. For example, customers '3' and '5' cannot form a cluster without incorporating '6,' neither can '3' and '6' form a cluster without incorporating '5' due to the presence of distribution network. However, '5' and '6' can form a cluster without incorporating '3' (as shown in yellow cluster). Nevertheless, '3,' '5,' and '6' independently can provide mitigation solutions to oneself without incorporating anyone else (see, red clusters). The pink cluster consisting of all the customers, as in Fig. 2(a), is also feasible.

In the current arrangement, like in an open system [29], [30], decision-making agents are simultaneously mitigation solution providers, allocators, and consumers. Electricity supply companies may also appropriate in this arrangement to attract future customers. To achieve the same, the customers and the electricity supply companies will be required to share the internal information to carry out the planning process. The methods of sharing internal information will be thoroughly discussed in Section III.C of this paper. In line with the analysis in [27], an appropriate and efficient utility distribution can only ensure "trustworthiness" [31] in this single shot game. However, consideration of multiple cluster formation may not result in the utility function to remain convex, and as a result, the core solution concept [32] may not exist. However, the solution concept based on an alternative definition of the core may exist [33]. Besides, utility distribution according to the "Fair allocation" solution concept, such as the nucleous [34], and the Shapley value [35] may exist as well.

Contributions of this paper are threefold:

i) The viability of a cluster set's existence relies on the relative location of all the customers and the topology of the distribution network itself. Therefore, the characteristics of such cluster formation strategies rely on graph-theoretic principles. A partitioning strategy has been developed in this regard. Since the occurrence of temporary faults within the cluster itself leads to vulnerability of the all the participants within a cluster, a fault rate threshold concept has been introduced to limit the cluster size.

ii) Contrary to traditional uniform utility distribution strategies, an alternative definition of core and "fair allocation" strategies has been considered in this work, which would ensure the formation of a socially acceptable outcome. Consideration of such a solution makes this work unique. Comparative analysis of three different socially acceptable outcomes will help the customers to decide the suitability of the considered allocation strategies.

iii) The participation in this CPR allocation problem is voluntary, while the solution provision among the customers is common. To avoid some of the customers' consequent free-riding status and resulting unilateral coalition deviation, one needs to carefully exchange willingness to pay information of the customers, especially when the game is of a single shot (planning problem). Strategies of sharing internal information among the contributors, along with associated advantages and disadvantages, have been discussed in detail.

The remainder of the paper is organized as follows. Calculation of the rivalrous component of CPR provision, the fault rate threshold, and the proposed graph-partitioning strategy are described in Section II. The theoretical development of the optimal contribution group formation, the optimization problem, and different strategies for sharing of such internal information are discussed in Section III. In Section IV, utility distribution strategies are compared for a five bus system with three customers, and, a eighteen bus system with eight customers. Sections V and VI contain discussion and concluding remarks, respectively.

II. Designing a Voltage Sag Mitigation Strategy

A. Aggregated Maximum Load Demand for DVR Sizing

If either a set of customers, an externally selected agent, or the electricity supply company, is installing a common sag mitigation device for multiple customers, it needs to be located in the distribution network itself. However, the customers are not the wire-owners of the distribution network. Additionally, it is to be noted that once the ratings of the DVR are decided, future expansion of loads in the downstream of the DVR would undermine such a sag mitigation solution provision. Therefore, assuming that the improved network performance will attract future customers, one needs to consider the participation of electricity supply companies in this CPR game. Like all other customers, the wire-owners (electricity supply companies) then need to disclose the projected maximum load demand completely, and this will also be considered as common knowledge. If the electricity supply company do not participate in this endeavor, they need to ensure that no additional customers are connected at the DVR downstream through an exclusivity agreement.

The customers may communicate the load profile (with the possibility of future expansion) to calculate the rivalrous component of the DVR provision. However, the DVR sizing based on the maximum load demand, considering future load expansion of the customers, would ensure that the DVR can meet the downstream load demand for all possible cluster formations, and all different operating conditions, and DVR will not be overloaded due to error in the estimation of customers load profile. Therefore, the maximum load demand has been utilized to calculate the current rating of the DVR. Since the rating of DVR can be obtained by multiplying associated current rating and voltage rating, the main objective of the paper is to calculate the voltage rating of the DVR, which is a (partially) non-excludable component of the CPR.

Since the distribution network is included within the clusters, distributed generators (DGs) either owned by customers or independent power producers located at the downstream of the DVR need to be accounted for calculating the rivalrous component. In this study, if the incorporation of DGs effectively decreases the maximum load demand of a customer, the presence of DGs needs to be accounted for in the procurement of a CPR resource.

B. Segregation of Sensitive and Non-Sensitive Component of Customers Loads

It is well known that the customers load demand constitutes of components that can either be sensitive or non-sensitive to voltage sags. The impact of voltage sags on both of these components can significantly differ. If the regulation permits, there exist two ways in which customers can express their marginal willingness to pay (if they want to be a part of a group):

i) The customers can bid as two different entities and declare the associated marginal willingness to pay separately. In this case, multiple customers' sensitive components may be protected by a common device, leaving their non-sensitive components behind. The enforcement of such a solution may not always be practical due to regulatory and feasibility challenges.

ii) Customers do not report the sensitive and non-sensitive components of the marginal willingness to pay separately; instead, they bid as a single entity. This classification is also applicable to the set of customers who cannot segregate the sensitive and non-sensitive components because of the physical connection. Since both the load components are protected by a DVR, while the protection of the sensitive components is only beneficial, customers maximum marginal willingness-to-pay, vis-á-vis the non-excludable component of the CPR provision (or simply, CPR provision) will be significantly reduced.

It is notable that this analysis can also be extended to a single customer problem with multiple sensitive devices and various different willingness to pay.

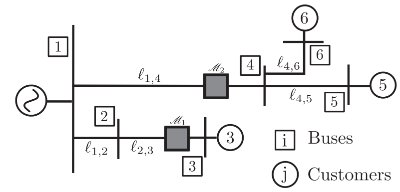

Fig. 3. Considering distribution lines as part of a cluster.

C. Fault Rate Threshold

The installation of sag mitigation devices does not affect the probability of the occurrence of faults within the distribution network. As indicated, a segment of the distribution network needs to be included within the cluster for devising a common solution. On the other hand, the players may choose not to include the distribution network segment and install mitigation devices on their own at an increased cost (because of not being able to utilize the benefit of economy of scale). In Fig. 3, $\mathscr{M}_1$ caters to the load '3,' and the load demands '5' and '6' are met through $\mathscr{M}_2$. With the installation of $\mathscr{M}_2$, the occurrence of a fault on the downstream distribution lines 4-5 and 4-6 would render both the cluster members '5' and '6' to become vulnerable. Here (in Fig. 3), i and j are bus and customer identifiers respectively. For simplicity, in subsequent figures with distribution networks, bus numbers are written within a box, while customer numbers are encircled.

Therefore, the contributors must mutually agree to establish a common fault rate threshold, $\mathcal{L}$, to allow the formation of clusters. The fault rate of a cluster, defined by the DVR, $\mathcal{M}_{(\cdot)}$, is given by the algebraic sum of the fault rate of all distribution lines, $\ell_{(\cdot,\cdot)}$ connected at the downstream of the DVR. A fault rate lower than a predetermined threshold selected by the customers would result in the viability of such cluster formation. With $O_{\mathcal{M}_{(\cdot)}}$ being the set of lines connected at the downstream of the

DVR, mathematically:

$$\mathcal{M}_{(\cdot)} = \sum_{\forall (q,r) \in O_{\mathcal{M}_{(\cdot)}}} \ell_{q,r} \leq \mathcal{L} \tag{1}$$

Consequently, the introduced fault rate threshold directly impacts the size of the cluster. If the fault rate of the distribution network branches is zero, incorporation of that branch within the cluster will not affect the contribution group size.

As a part of the fuse-saving reclosing operation, the reclosers will try to isolate the fault first. Also, the customers experiences the sag before the operation of fuses/reclosers. If the fault occurs at the downstream of the DVR, it is well known that DVR will not be able to protect corresponding customers during the sags independent of the recloser's location. If the fault occurs at the upstream, DVR will duly inject the desired voltage to the customers experiencing the sag. Therefore, it is noted that the DVR's capability to mitigate voltage sags is determined by their location and not by the recloser's location. Nevertheless, reclosers and associated fuse-recloser coordination settings will determine the duration of voltage sags. Since customers do not have a say on locations of fuses/reclosers in the network and mitigation of voltage sag magnitude is the main focus of this paper, the impact of reclosers is not accounted for in this work.

D. Graph-Partitioning Strategy

A generic radial distribution network clustering problem for the series-connected mitigation devices allocation is given as follows:

Definition 1: Given an undirected rooted tree $G_{Ro} = (\mathrm{V}, \mathrm{E})$, with the set of root nodes and leaf nodes being V_R and V_{leaf}, respectively, it is aimed to find a collection of connected subsets of nodes (which may also be called as clusters), $\mathrm{V}_1, \mathrm{V}_2, \ldots, \mathrm{V}_{ic}, \ldots, \mathrm{V}_g, \ldots$, so that the following conditions are satisfied.

1) $\mathrm{V}_{\text{leaf}} \subseteq \cup_{\forall ic} \mathrm{V}_{ic}$ (each leaf node is a part of some cluster).
2) $\mathrm{V}_{\text{leaf}} \cap \mathrm{V}_{ic} \cap \mathrm{V}_g = \varnothing; \quad \forall_{ic \neq g}$ (each leaf node is a part of one and only one cluster).
3) $G_{Su}[\mathrm{V}_{ic}]$, the subgraph of G_{Ro}, induced by the set of nodes, V_{ic}, connected by the subset of the edges, E, remains connected.
   a) If $|\mathrm{V}_{ic}| \geq 1$, then $\mathrm{V}_{ic} \cap \mathrm{V}_{\text{leaf}} \neq \varnothing$ (if a cluster V_{ic} is non-null, then it must contain at least one leaf node, where $|(\cdot)|$ calculates the cardinality of a set).
   b) If a subgraph $G_{Su}[\mathrm{V}_{ic}]$ includes the root node, V_R, and the degree of the root node in the subgraph is equal to the degree of the root node in the rooted tree, G_{Ro}, there will be no vertices within the subgraph, G_{Su}, the degree of which will be different compared to the degree of equivalent vertices of the rooted tree, G_{Ro}. Else, there will be at most one vertex in the subgraph, G_{Su}, the degree of which will be different compared to the degree of equivalent vertices of the rooted tree, G_{Ro}. The degree of a node in a graph can be defined as the number of edges connected to it. ■

In this work, the root node of the discussed tree structure in Definition 1 represents the substation. The customers are located at the leaf nodes. Therefore, the number of clusters is limited by the number of customers or the number of leaf nodes.

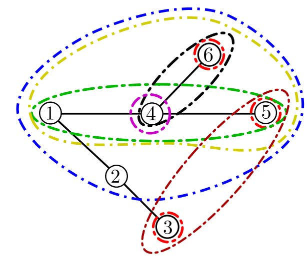

Fig. 4. Cluster formation in a distribution network.

The proposed graph-partitioning strategy (Definition 1) has been explained using the sample six node tree given in Fig. 4. This tree structure represents the graph of the network shown in Fig. 3 (which is adapted from the network shown in Fig. 2(b)). In the figure, node '1' is the root node, and nodes '3,' '5' and '6' are the leaf nodes. Nodes '2' and '4' are intermediate nodes that connect the root node with the leaf nodes. Dotted regions signify different clusters.

Red clusters in Fig. 4 are always feasible because they include only one leaf node, i.e., a customer. For simplicity, it has been assumed that each customer is supplied through at most one DVR (Multi-DVR cascading is not accounted here; and can be easily incorporated). This assumption provides us with the second condition, wherein, each of the leaf nodes is part of only one cluster. Depending upon the configuration, the DVRs can improve the voltage sag performance only at the downstream buses. Therefore, all the nodes within a cluster must stay connected. For example, each of the gold, black, and red clusters, shown in Fig. 4, are connected.

Since the DVR mitigates voltage sags in all the downstream nodes of a tree, given a cluster, only one node in it, especially where the DVR is connected, can have a degree not equal to that of the original tree. However, in the green cluster, both nodes '4' and '1' are of degree less than that of the original cluster, signifying the installation of the DVR at node '1' improves the voltage sag performance of node '5,' while ignoring the existence of node '6.' This condition is not feasible. Although the purple cluster encircles only node '4', and subsequently contains exactly one vertex with degree different to that of the rooted tree, it does not contain any leaf node, resulting into its impracticality. The maroon cluster includes nodes '3' and '5', where, both these nodes of of different degree compared to original rooted tree, indicating its infeasibility. In the blue cluster, the DVR is installed at the substation, incorporating node '2,' but does not include node '3.' This condition is also not viable.

Remark 1: Any two clusters cannot have more than one element in common. If either of them is singleton or null-set, they will have no elements in common. Mathematically, $0 \leq |\mathrm{V}_{ic} \cap \mathrm{V}_g| \leq 1, \forall_{ic \neq g}$. Given the connectedness of the rooted tree, condition 3b ensures that none of the feasible clusters is disjoint.

III. Optimal CPR Group Formation

In the considered CPR provision problem, the objectives are twofold: (i) which contribution group generates how much of non-excludable CPR provision and associated cluster formation, and, (ii) how the total expense will be shared among the contributors. Our objective will be to find cluster set generating the highest utility; where, the utility being shared is also socially appropriate.

While the introduced fault rate threshold can limit the size of the cluster, here, the impact of sags on the customers is indirectly accounted for in the customers willingness to pay function. It is assumed that the customers capture the cost of voltage sags through detailed simulation, and bid their marginal willingness to pay. Furthermore, as in [27], it is considered that customers' marginal willingness to pay is linearly decreasing. Calculation of non-excludable CPR provision and utility generated by each cluster can be calculated using the following definition, which is partially obtained from [27], is given below:

Definition 2: Let the CPR cluster, g, is formed by utilizing a finite set of nodes, V_g, where, for any node $q \in \mathrm{V}_g$ (assuming all the nodes may not be occupied by contributors or players), $\beta_q \geq 0$ be the rivalrous part, and $\alpha \geq 0$ be non-excludable part of consumption, such that the total consumption of each player q is given by $\alpha\beta_q\ (>0)$. Any player's maximum non-excludable CPR requirement is given by α_{MAX} and the marginal willingness to pay with respect to the non-excludable part of its CPR provision is given by $a_q(1-\alpha/\alpha_{MAX})$, where $a_q \geq 0$. Beyond $\alpha = \alpha_{MAX}$ the willingness to pay is undefined. If a node, q, is not occupied by any contributor, or, player, associated a_q and β_q are zero. Under voluntary participation, let all the players declare their marginal willingness to pay. Also, let the average cost of production be given by a linear function, $\sum_{\forall q\in \mathrm{V}_g}\beta_q(c - d\alpha\sum_{\forall q\in \mathrm{V}_g}\beta_q)$, such that $c>0$, $d\geq 0$ and $\sum_{\forall q\in \mathrm{V}_g}\beta_q > 0$ (note the strictness). Beyond $\alpha = \frac{c}{2d\sum_{\forall q\in \mathrm{V}_g}\beta_q}$, the cost of production is also undefined. Then optimal CPR provision for the cluster g, α^g_{MKT}, and the utility generated Γ^g can be given by,

$$\alpha^g_{MKT} = \begin{cases} \alpha_{MAX}\dfrac{\sum_{\forall q\in \mathrm{V}_g} a_q - c\sum_{q\in \mathrm{V}_g}\beta_q}{\sum_{\forall q\in \mathrm{V}_g} a_i - 2\,d\alpha_{MAX}\left(\sum_{q\in \mathrm{V}_g}\beta_q\right)^2} \\ \text{if}\quad \sum_{\forall q\in \mathrm{V}_g} a_q \geq c\sum_{\forall q\in \mathrm{V}_g}\beta_q \\ \geq 2\,d\alpha_{MAX}\left(\sum_{\forall q\in \mathrm{V}_g}\beta_q\right)^2 > 0 \\ \dfrac{c}{2d\sum_{\forall q\in \mathrm{V}_g}\beta_q}\text{if}\quad \sum_{\forall q\in \mathrm{V}_g} a_q \geq \dfrac{c\sum_{\forall q\in \mathrm{V}_g}\beta_q}{2-\dfrac{c\sum_{\forall q\in \mathrm{V}_g}\beta_q}{2d\alpha_{MAX}\left(\sum_{\forall q\in \mathrm{V}_g}\beta_q\right)^2}} \\ \text{and}\quad 0 < c\sum_{\forall q\in \mathrm{V}_g}\beta_q < 2\,d\alpha_{MAX}\left(\sum_{\forall q\in \mathrm{V}_g}\beta_q\right)^2 \\ 0 \qquad \text{otherwise} \end{cases} \tag{2}$$

$$\Gamma^g = \begin{cases} \frac{1}{2}\alpha_{MAX}\dfrac{\left(\sum_{\forall q\in \mathrm{V}_g} a_q - c\sum_{\forall q\in \mathrm{V}_g}\beta_q\right)^2}{\sum_{\forall q\in \mathrm{V}_g} a_q - 2\,d\alpha_{MAX}\left(\sum_{\forall q\in \mathrm{V}_g}\beta_q\right)^2} \\ \text{if}\quad \sum_{\forall q\in \mathrm{V}_g} a_q \geq c\sum_{\forall q\in \mathrm{V}_g}\beta_q \\ \geq 2\,d\alpha_{MAX}\left(\sum_{\forall q\in \mathrm{V}_g}\beta_q\right)^2 > 0 \\ \dfrac{c\sum_{\forall q\in \mathrm{V}_g} a_q}{2d\sum_{\forall q\in \mathrm{V}_g}\beta_q}\left(1-\dfrac{c}{4\,d\alpha_{MAX}\sum_{\forall q\in \mathrm{V}_g}\beta_q}\right) - \dfrac{c^2}{4\,d} \\ \text{if}\quad \sum_{\forall q\in \mathrm{V}_g} a_q \geq \dfrac{c\sum_{\forall q\in \mathrm{V}_g}\beta_q}{2-\dfrac{c\sum_{\forall q\in \mathrm{V}_g}\beta_q}{2d\alpha_{MAX}\left(\sum_{\forall q\in \mathrm{V}_g}\beta_q\right)^2}} \\ \text{and}\quad 0 < c\sum_{\forall q\in \mathrm{V}_g}\beta_q < 2\,d\alpha_{MAX}\left(\sum_{\forall q\in \mathrm{V}_g}\beta_q\right)^2 \\ 0 \qquad \text{otherwise} \end{cases} \tag{3}$$

■

Here, the CPR provision and the utility generation corresponding to the condition, $c \geq 2\,d\alpha_{MAX}\sum_{\forall q\in \mathrm{V}_g}\beta_q$, are obtained from [27]. If, $c < 2\,d\alpha_{MAX}\sum_{\forall q\in \mathrm{V}_g}\beta_q$, CPR will still be provided, if, $\sum_{\forall q\in \mathrm{V}_g} a_q \geq \frac{c\sum_{\forall q\in \mathrm{V}_g}\beta_q}{2-\frac{c\sum_{\forall q\in \mathrm{V}_g}\beta_q}{2d\alpha_{MAX}\left(\sum_{\forall q\in \mathrm{V}_g}\beta_q\right)^2}}$, and associated CPR provision will be $\frac{c}{2\,d\sum_{\forall q\in \mathrm{V}_g}\beta_q}$ (the limiting value). The utility function will be calculated accordingly.

It is imminent that in the DVR allocation problem, α_{MAX} will be unity. Here, a_q of the marginal willingness to pay function will be unique for each customer and will be a function of network topology and fault-rates, the customers' location within the network, and their internal information. As discussed earlier, a customer may segregate its demand and may bid as multiple entities through different a_q.

Instead of utilizing Definition 2, one can also solve the exact optimization problem for calculating the optimal utility function for each of the cluster.

A. *Utility Generated by Various Contribution Sets*

It is imminent that V_{leaf} is the set of all players in the CPR provisioning problem. Depending upon the set of players forming the contribution group S, $(\subseteq \mathrm{V}_{\text{leaf}})$, where all the players in the group S contribute, players may divide themselves into several clusters. The decision variable symbolizing whether a player/node, q, joins a cluster, g, or not, is given by $\chi^{g,S}_q$, as follows:

$$\chi^{g,S}_q = \begin{cases} 1 \text{ if node } q \text{ in contribution set } S \text{ joins cluster } g \\ 0 \qquad\qquad \text{otherwise} \end{cases} \tag{4}$$

Given a set of contributors, S, optimum utility generated by a cluster g, $\Gamma^{g,S}$ (given in (8) shown at the bottom of the next page), and optimal CPR provision, $\alpha^{g,S}$, can be calculated using Definition 2 (in this paper). For a contribution group, S, that intends to maximize its net utility over all possible cluster sets, the optimization problem can be given below:

$$\Gamma^S = \max\sum_{\forall g}\Gamma^{g,S} \tag{5}$$

Here, V_g is the selected set of nodes encapsulated by the cluster g, subject to constraints of the graph partitioning principle (given in Definition 1) and the fault rate threshold. According to the conditions 1 and 2 (of the group partitioning strategy) if $\mathcal{Y}^S_q$ depicts the contribution status of the player q, in group S, then:

$$\sum_{\forall g}\chi^{g,S}_q = \mathcal{Y}^S_q; \quad \forall q \in \mathrm{V}_{\text{leaf}} \tag{6}$$

To identify the nullity of a given cluster (say, g), for a given contribution group (say, S), the following conditions are used:

$$U^{g,S} = \begin{cases} 1 \text{ if } \sum_{\forall q\in \mathrm{V}_g}\chi^{g,S}_q \leq 0 \\ 0 \text{ if } \sum_{\forall q\in \mathrm{V}_g}\chi^{g,S}_q > 0 \end{cases}; \quad \forall g \tag{7}$$

According to the condition 3a, a non-null cluster must contain at least one leaf node,

$$\sum_{\forall q\in \mathrm{V}_{\text{leaf}}}\chi^{g,S}_q \geq 1 - U^{g,S}; \quad \forall g \tag{9}$$

The degree of a given node (say, q), in a given cluster (say, g), in a given contribution group (say, S), $\mathfrak{D}_q^{g,S}$, can be given by,

$$\mathfrak{D}_q^{g,S} = \sum_{\forall r, r \neq q} \chi_q^{g,S} \chi_r^{g,S} \mathfrak{B}_{q,r}; \quad \forall g, q \tag{10}$$

Following conditions are used to determine whether the degree of node q of cluster g in the contribution set S is equal to the degree of the rooted tree's corresponding node:

$$W_q^{g,S} = \begin{cases} 1 & \text{if } \mathfrak{D}_q^{g,S} = \mathcal{D}_q \\ 0 & \text{if } \mathfrak{D}_q^{g,S} < \mathcal{D}_q \end{cases}; \quad \forall g, q \tag{11}$$

The following criterion needs to be satisfied for fulfilling condition 3b:

$$\sum_{\forall q \in \mathrm{V}_g} \left(1 - W_q^{g,S}\right) \chi_q^{g,S} \leq 1 - \sum_{\forall r \in \mathrm{V}_R} W_r^{g,S} \chi_r^{g,S}; \quad \forall g \tag{12}$$

where, V_R is the set of root nodes. If the equivalent fault rate of distribution lines are $\ell_{q,r}$, and as agreed upon by the cluster members, if the internal length of each cluster (fault rate threshold) is required to be limited to $\mathcal{L}$, then,

$$\sum_{\forall q, r, r \neq q} \chi_q^{g,S} \chi_r^{g,S} \ell_{q,r} \leq \mathcal{L}; \quad \forall g \tag{13}$$

The optimization problem (5)–(13) is combinatorial in nature, which is solved using the solver Couenne [36] in General Algebraic Modeling System (GAMS) development environment. Given a set of players, the optimization problem (5)–(13) needs to be solved for all possible contribution groups.

Remark 2: Free-riders with zero contribution may be incorporated in the optimal group if and only if their incorporation actively improves the group's utility.

Theorem 1: Utility generated by different contribution groups is super-additive.

Proof: From Remark 2, the contribution group will add a new player into a cluster if such a contributor does not decrease the group's net utility. ■

Fact 1: Players in a super-additive game are always incentivized to form a grand coalition.

B. Utility Distribution Among the Contributors

1) Alternative Definition of Core Allocation Strategy: Although possible utility function may render the core solution concept to become empty, the core may exist if one focuses on the deviating group, S, and its subsets. The alternative definition of the core [33] is given by,

$$\mathscr{O}(S) = \{(\alpha, u) : \sum_{\forall q \in T} u_q \mathcal{Y}_q^T \geq \Gamma^T, \forall T \subseteq S\} \tag{14}$$

In the current context, it is intended to find the group generating maximum utility while also being immune from further deviations. Here, Γ^T is obtained by solving (5)–(13). The optimal deviating group, in this case, can be obtained by solving the following optimization problem:

$$\max \sum_{\forall S \in \Omega} \Gamma^S A^S \tag{15}$$

subject to,

$$\sum_{\forall S \in \Omega} A^S = 1; \quad A^S \in \{0, 1\} \tag{16}$$

$$\sum_{\forall S \in \Omega} \sum_{\forall q \in \mathrm{V}_{\text{leaf}}} u_q \mathcal{Y}_q^S A^S = \sum_{\forall S \in \Omega} \Gamma^S A^S \tag{17}$$

$$\sum_{\forall S \in \Omega} \mathfrak{T}^{S,T} A^S \sum_{\forall q \in \mathrm{V}_{\text{leaf}}} u_q \mathcal{Y}_q^T + M \left(1 - \sum_{\forall S \in \Omega} \mathfrak{T}^{S,T} A^S\right) \geq \Gamma^T; \quad \forall T \subseteq S \backslash \varnothing \tag{18}$$

where, A^S is a binary variable identifying whether the group S generates the maximum utility and (16) enforces that such condition exists for only one of such groups. Equations (17) and (18) indicate that the utility distribution under group S is better compared to all possible subgroups T ($\subseteq S$). Additionally, Ω is the power set constituting of all possible combinations of V_{leaf} (the contributor set). The combinatorial optimization problem (15)–(18) is solved using the solver SCIP [36] in GAMS. Like the core solution concept, possible utility distribution strategies in this alternative definition can be infinite. So, the main aim is to identify the optimal contribution group. Equations (5)–(13) will provide the optimal set of clusters for the optimal contribution group.

2) Fair Allocation: Two different fair allocation strategies, namely, the nucleous and the Shapley value, are considered in this paper. In contrast to the alternative definition of the core, the nucleous and Shapley value are unique, and the nucleous always exists for a game with non-empty imputation.

i) The nucleous: The nucleous essentially tries to find an imputation that minimizes the participants' worst dissatisfaction and can be obtained by solving a series of linear programs (LP). The detailed definition of the nucleous can be found in [34].

ii) The Shapley value: Depending upon the marginal contribution of each player, Shapley proposed a solution concept [35] that focuses on the relative importance of the players in the game.

$$\Gamma^{g,S} = \begin{cases} \frac{1}{2}\alpha_{MAX} \frac{\left(\sum_{\forall q \in \mathrm{V}_g} a_q \chi_q^{g,S} - c \sum_{\forall q \in \mathrm{V}_g} \beta_q \chi_q^{g,S}\right)^2}{\sum_{\forall q \in \mathrm{V}_g} a_q \chi_q^{g,S} - 2\, d\alpha_{MAX} \left(\sum_{\forall q \in \mathrm{V}_g} \beta_q \chi_q^{g,S}\right)^2} \\ \text{if } \sum_{\forall q \in \mathrm{V}_g} a_q \chi_q^{g,S} > c \sum_{\forall q \in \mathrm{V}_g} \beta_q \chi_q^{g,S} \geq 2\, d\alpha_{MAX} \left(\sum_{\forall q \in \mathrm{V}_g} \beta_q \chi_q^{g,S}\right)^2 > 0 \\ \frac{c \sum_{\forall q \in \mathrm{V}_g} a_q \chi_q^{g,S}}{2d \sum_{\forall q \in \mathrm{V}_g} \beta_q \chi_q^{g,S}} \left(1 - \frac{c}{4\, d\alpha_{MAX} \sum_{\forall q \in \mathrm{V}_g} \beta_q \chi_q^{g,S}}\right) - \frac{c^2}{4\, d} \\ \text{if } \sum_{\forall q \in P} a_q \chi_q^{g,S} \geq \frac{c \sum_{\forall q \in \mathrm{V}_g} \beta_q \chi_q^{g,S}}{2 - \frac{c \sum_{\forall q \in \mathrm{V}_g} \beta_q \chi_q^{g,S}}{2d\alpha_{MAX} \left(\sum_{\forall q \in \mathrm{V}_g} \beta_q \chi_q^{g,S}\right)^2}} \text{and } 0 < c \sum_{\forall q \in \mathrm{V}_g} \beta_q \chi_q^{g,S} < 2\, d\alpha_{MAX} \left(\sum_{\forall q \in \mathrm{V}_g} \beta_q \chi_q^{g,S}\right)^2 \\ 0 \qquad \text{otherwise} \end{cases} \tag{8}$$

The value received by each player can be given by,

$$\mathrm{Phi}_q = \sum_{\forall S \subset \mathrm{V}_{\mathrm{leaf}}} \frac{(|S|-1)!\,(|\mathrm{V}_{\mathrm{leaf}}|-|S|)!}{|\mathrm{V}_{\mathrm{leaf}}|!} \left[\Gamma^{S} - \Gamma^{S\setminus\{q\}}\right] \quad (19)$$

C. Strategies for Sharing of Internal Information

One may treat the CPR resource provision in this problem as an open system, where the DVR customers (including electricity supply company) are the appropriators, allocators, and consumers. Non-repeatability of the considered game suggests the existence of free-riders. In the event one argues that the customers are unwilling to share information about their internal willingness-to-pay function among each other, they may ask an external agent to carry out a similar exercise on their behalf while following the regulatory guidelines. The customers and the electricity supply companies will disclose their internal willingness-to-pay function simultaneously (as per [27]), and the external agent will impartially solve the proposed optimization problems and indicate the optimal location of mitigation solutions. Based on the derived solution by the external agent, each of the participants will be needed to appropriate to recover the cost of installation and maintenance of the mitigation solution. The external agent will have the legal right to enforce DVR customers to appropriate in the mitigation solution.

In this regard, it is important to note that once the corresponding customers willingness-to-pay is disclosed, they are legally bound to appropriate. Only the customers strictly improving the utility of the group will be a part of a valid cluster. Due to partial excludability, non-contributors may be eliminated, and their unilateral deviational utility can become zero. This way, all the customers and the electricity supply companies have a strong incentive to appropriate (and, not free-ride) in the CPR. Since the contribution to the resource provision is based on the declared internally calculated willingness-to-pay functions, unless the external agent knows the true willingness-to-pay for participants (customers or electricity supply companies), it wouldn't be able to know their individual free-riding status.

The electricity supply company is unable to carry out this planning exercise since it is one of the partners investing in the common voltage sag mitigation solution. If all the customers report their internal information to the electricity supply companies, they themselves may falsely represent its internal information and procure the mitigation solution at a lower cost. Given the regulatory guideline, if electricity supply companies sell the mitigation solution provision at the procurement cost; the new customers will have an unfair advantage in this endeavor. While one may argue that similar exercise can be carried out and priced by the electricity supply company; in such a case, the good will cease to be a CPR. The solution to such kind of problem will be a part of future work. Therefore, the electricity supply company will only be allowed to carry out this planning exercise if it does not participate.

In the considered CPR sharing problem, voluntary participation is desired to avoid socially inefficient outcome. However, although a utility reducing non-contributor will not be a part of the cluster, unless proven otherwise, free-riding provision exists if a non-contributing customer increases the total utility of the group. Therefore, the internal information sharing strategy's primary objective is to prohibit sub-group coalition deviation among the customers and associated free-riding. Therefore, the proposed information exchange mechanism complements the proposed CPR formation and utility distribution strategies.

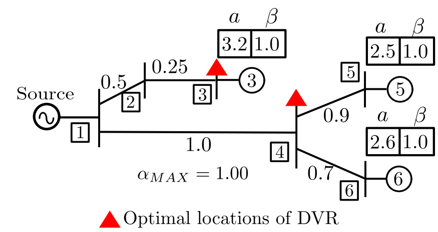

Fig. 5. Optimal set of clusters in the considered distribution network.

TABLE I
GENERATED CPR AND UTILITY FOR DIFFERENT CONTRIBUTION GROUPS

Contribution Group	CPR Generated			Utility Generated
	'3'	'5'	'6'	
'6'	0.0000	0.0000	0.2326	0.0698
'5'	0.0000	0.2016	0.0000	0.0504
'6','5'	0.0000	0.2191	0.2191	0.1205
'3'	0.3774	0.0000	0.0000	0.2264
'3','6'	0.3774	0.0000	0.2326	0.2962
'3','5'	0.3774	0.2016	0.0000	0.2768
'3','5','6'	0.3774	0.2191	0.2191	0.3469

IV. Case Study

While the scope of this section is limited to determining the optimal capacity requirements and utility distribution with the voltage sag mitigation solution with DVRs, the proposed methodology can be utilized in any system that follows similar characteristics. In line with the voltage sag mitigation application, α_{MAX} is selected to be 1.00 in one of the examples. While in another example, α_{MAX}, is selected to be 1.08 to show the genericness of the methodology. Consideration of three customers in the first two examples ensure numerical verification of the solution of the optimization problem. A relatively larger distribution network is also considered in the third example, to show the effectiveness of the proposed methodology.

Fig. 5 shows a typical four bus radial network, along with the discussed requisite parameters. The parameters of the distribution network in terms of the fault rate of each of the lines in the distribution network are provided. The maximum marginal willingness to pay of each customer and their maximum load demand as a part of its bid is also provided. The production cost function is considered to be linear (within its viable operating region), and the economy of scale exists throughout the production cost curve. The parameters c and d, representing the production cost, are 2.00 and 0.01, respectively. Additionally, the fault rate threshold, $\mathcal{L}$, is selected to be 4.0 and 2.0 for parametric analysis.

Table I shows the non-excludable component of the CPR and maximum utility generated by different contribution groups that are calculated by solving the optimization problem defined in (5)–(13). It can be observed that when either of the customers (each node represents only one customer) '3' and '5' is the only provider, even if the economy of scale exists and $\mathcal{L} = 4$, '3' and '5' would not like to incorporate '6' in the formation of the CPR, respectively, eliminating free-riders. The solution here is independent of $\mathcal{L} = 2$ and 4.

TABLE II
MATHEMATICALLY CALCULATED UTILITY DISTRIBUTION

Solution Concepts	u_3	u_5	u_6
Nucleous	0.2264	0.0507	0.0698
The Shapley Value	0.2264	0.0506	0.0699

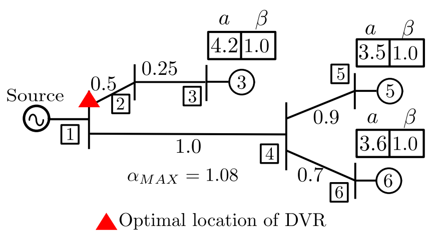

Fig. 6. Optimal set of clusters in the considered distribution network with parameters of different customers.

It is also seen that when the customers '5' and '6' are in the contribution group and fault rate threshold permits (fault rate threshold are satisfied in both cases of the considered parametric analysis), these customers form a common cluster. Even though the non-excludable CPR received by the customer '6' declines, the combined total utility received is increased by 0.0003 units. However, when all the three players are contributors, incorporation of '3' into the cluster '5,' '6' or vice-versa reduces the total utility, leading to the formation of multiple clusters, one containing customers in '3' only and the other containing customers in both '5' and '6.'

Table I also indicates that the utility generated is supermodular, and hence the core solution concept exists, and the grand coalition satisfies alternative core definition. The utility distribution according to the core can be given by,

$$\begin{aligned}\mathscr{C}(\text{`3,'`5,'`6'}) = \{&u \in \mathbb{R}^3_+ : u_3 + u_5 + u_6 = 0.3469,\\ &u_3 + u_5 \geq 0.2768, u_3 + u_6 \geq 0.2962,\\ &u_5 + u_6 \geq 0.1205,\ u_3 \geq 0.2264,\\ &u_5 \geq 0.0504,\ u_6 \geq 0.0698\}\end{aligned} \tag{20}$$

The solution for (20) contains mathematically calculated solution $u_3 = 0.2264$, $u_5 = 0.0504$, $u_6 = 0.0701$, obtained by solving (15)–(18), the alternative definition of the core (among many possible solutions). The utility distribution, according to the 'Fair allocation' strategy, is given in Table II.

The calculated Shapley value and nucleous are found to be lying within the core. The optimal non-excludable CPR provision (common voltage injected by the DVR) corresponds to the grand coalition in Table I. The utility of individual customers is given by subtracting individual appropriation from perceived benefit. Given the optimal CPR provision, the total perceived benefit can be directly calculated from the linear willingness to pay function. Therefore, given the utility distribution and the perceived benefit, individual investment cost can be indirectly calculated.

The optimal set of clusters with a similar four-bus radial network with different appropriation levels is given in Fig. 6. To ensure the genericness of our approach, in this case, α_{MAX} is selected to be 1.08 and $\mathcal{L}$ is selected to be 4.0.

In this case as well, it has been found that the utility function is supermodular, and hence the core exists. Unlike the previous case, the use of one mitigation device is cost-optimal. If the fault rate threshold does not limit the cluster size (which is the case here), the utility distribution according to the core solution concept will be:

$$\begin{aligned}\mathscr{C}(\text{`3,'`5,'`6'}) = \{&u \in \mathbb{R}^3_+ : u_3 + u_5 + u_6 = 1.3659,\\ &u_3 + u_5 \geq 0.9748,\ u_3 + u_6 \geq 1.0118,\\ &u_5 + u_6 \geq 0.7356,\ u_3 \geq 0.6255,\\ &u_5 \geq 0.3493,\ u_6 \geq 0.3863\}\end{aligned} \tag{21}$$

As indicated earlier, a similar problem has been solved for a larger network with an increased number of buses and is shown in Fig. 7. The CPR generated, and the utility distribution for various customer groups is also given in Table III.

TABLE III
UTILITY DISTRIBUTION WITH EIGHT CUSTOMERS

Customer Index	CPR Generated	Utility Distribution	Shapley Value
5	0.3773	0.2264	0.2263
6	0.2191	0.0698	0.0702
7	0.2191	0.0508	0.0508
9	0.5025	0.5025	0.5028
14	0.6689	1.3378	1.3375
15	0.3773	0.2264	0.2261
17	0.2191	0.0698	0.0697
18	0.2191	0.0508	0.0508

While the supermodularity ensures the core and the grand coalition's existence, one may observe that the optimal solution does not include all the customers as a part of a common DVR. Rather the cluster formation stems from the condition that if one customer is not beneficial for the group, it will be discarded. Nevertheless, in the given cases, the analysis indicates that the utility distribution according to either of the considered three methodologies would lead to a socially justifiable outcome.

V. DISCUSSION

Although the proposed methodology actively discourages free-riders participation by strategically excluding them for avoiding socially unjustifiable outcome, this method suffers from the following numerical complexities.

i) Although some of the conditional statements in (5)–(13) involve the multiplication of binary variables, most of the non-linearities can be suitably linearized. However, the resulting optimization problem remains an MINLP problem, which can be challenging to solve. The non-linearity stems from the Definition 2, which can be alleviated through certain precalculation.
ii) Equilibrium CPR provision and utility generation for different contribution groups need to be calculated independently. However, with an increasing number of participants, the number of different contribution groups for which these two quantities need to be calculated grows following a geometric progression. Besides, exponentially growing binary variables imposes an additional burden.
iii) The alternative Core allocation strategy also introduces binary variables. However, the resulting formulation is a mixed-integer programming (MIP) problem. The number of binary variables in the optimization problem also exponentially increases with an increasing number of participants.

Accordingly, the proposed algorithm is non-polynomial in complexity. However, because the considered problem is a

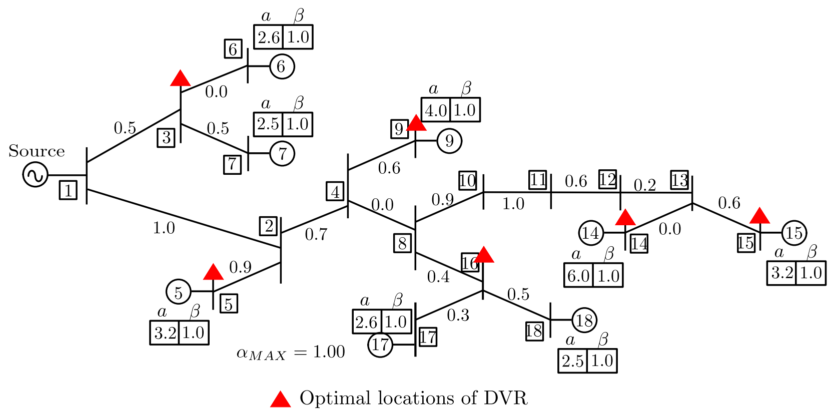

Fig. 7. Optimal set of clusters in the considered larger distribution network with various system parameters.

planning problem, the computational burden is undoubtedly not a constraint. Also, because equilibrium CPR provisions for different contribution groups are independent, the solution time can be significantly improved by employing parallel computing. Furthermore, Theorem 1 also provides us with a feasible initial solution, further reducing the computation time. As for the computational time, the first two examples, involving three customers in a six node system, each requires 30 minutes of CPU time, while for the third example, involving eight customers in an eighteen node system, the total CPU time has increased to approximately ten hours. We have utilized parallel computing in the third example. All three examples are solved in an Intel i7-based workstation with 32 GB of RAM.

VI. Conclusion

In this paper, the provision of multiple contribution group formation for the optimal voltage sag mitigation using DVRs has been discussed. While incurring costs for designing and installing DVR, the existence of the economy of scale is a motivating factor for the common mitigation solution provision. Here, voltage sag mitigation using DVRs has been classified as a CPR with partial excludability. A graph-partitioning principle has been proposed to obtain multiple feasible contribution cluster sets, where a common DVR will serve each of the clusters. Because a portion of fault-prone distribution network needs to be incorporated within the cluster, the customers may limit their participation in the group based on the introduced fault rate threshold. Furthermore, the customers' willingness to pay is utilized to calculate optimal CPR and utility distribution for different contribution group. However, the appropriate distribution of utility generated by different contribution groups is essential for the contributing members to remain within the group. In this regard, three different solution approaches, such as the alternative core definition, the nucleous, and the Shapley value, have been considered.

The proposed utility generation and distribution rely on the willingness-to-pay for each of the customers, thus eliminating mandatory participation and contribution requirements for the customers with zero willingness-to-pay. The partial excludability ensures that utility-reducing and unilaterally deviant customers may not be included in any contribution group, eliminating free-riding. The considered information sharing strategy also upholds the same and ensures voluntary participation. While the scope of the proposed work can be diverse, the methodology has been numerically verified utilizing a small scale network for voltage sag mitigation as an example. A larger network has also been considered to demonstrate the effectiveness of the proposed methodology. One of the main disadvantages of the proposed methodology is the large computational complexity imposed by mixed-integer solution space. However, the computational time can be reduced by applying one of the analytical results and utilizing parallel computing. Nevertheless, being a planning problem, the bottleneck in terms of computational complexity is not a major concern.

Acknowledgment

The authors would like to thank Ministry of Human Resource and Development (MHRD), India and University Postgraduate Award (UPA), and Institute Postgraduate Tuition Award (IPTA) from University of Wollongong, Australia for the financial support.

References

[1] M. McGranaghan and B. Roettger, "Economic evaluation of power quality," *IEEE Power Eng. Rev.*, vol. 22, no. 2, pp. 8–12, Feb. 2002.

[2] V. K. Ramachandaramurthy, A. Arulampalam, C. Fitzer, C. Zhan, M. Barnes, and N. Jenkins, "Supervisory control of dynamic voltage restorers," *IET Gener. Transmiss. Distrib.*, vol. 151, no. 4, pp. 509–516, Jul. 2004.

[3] M. J. Sullivan, T. Vardell, and M. Johnson, "Power interruption costs to industrial and commercial consumers of electricity," *IEEE Trans. Ind. Appl.*, vol. 33, no. 6, pp. 1448–1458, Nov./Dec. 1997.

[4] Z. Hanzelka and A. Baggini, "Voltage dips and short supply interruptions," in *Handbook of Power Quality,* A. Baggini, Ed., Chichester, U.K.: Wiley, 2008, ch. 4, pp. 79–134.

[5] J.-H. Lee, S.-G. Jeon, D.-K. Kim, J.-S. Oh, and J.-E. Kim, "Temporary fault ride-through method in power distribution systems with distributed generations based on PCS," *Energies*, vol. 13, no. 5, Mar. 2020, Art. no. 1123.

[6] M. H. J. Bollen, *Understanding Power Quality Problems: Voltage Sags and Interruptions*. New York, NY, USA: Wiley, 2000.

[7] J. G. Nielsen and F. Blaabjerg, "A detailed comparison of system topologies for dynamic voltage restorers," *IEEE Trans. Ind. Appl.*, vol. 41, no. 5, pp. 1272–1280, Sep./Oct. 2005.

[8] A. Sannino, M. G. Miller, and M. H. J. Bollen, "Overview of voltage sag mitigation," in *Proc. IEEE Power Eng. Soc. Winter Meeting Conf.*, Jan. 2000, pp. 2872–2878.

[9] F. C. Pereira, O. C. N. Souto, J. C. De Oliveira, A. L. A. Vilaca, and P. F. Ribeiro, "An analysis of costs related to the loss of power quality," in *Proc. 8th Int. Conf. Harmon. Qual. Power*, Oct. 1998, pp. 777–782.

[10] J. V. Milanovic and C. P. Gupta, "Probabilistic assessment of financial losses due to interruptions and voltage sags–Part I: The methodology," *IEEE Trans. Power Del.*, vol. 21, no. 2, pp. 918–924, Apr. 2006.

[11] Y. Zhang and J. V. Milanovic, "Global voltage sag mitigation with facts-based devices," *IEEE Trans. Power Del.*, vol. 25, no. 4, pp. 2842–2850, Oct. 2010.

[12] J. V. Milanovic and Y. Zhang, "Global minimization of financial losses due to voltage sags with FACTS based devices," *IEEE Trans. Power Del.*, vol. 25, no. 1, pp. 298–306, Jan. 2010.

[13] N. G. Hingorani, "Introducing custom power," *IEEE Spectr.*, vol. 32, no. 6, pp. 41–48, Jun. 1995.

[14] R. Mitra, A. K. Goswami, and P. K. Tiwari, "Optimal selection of voltage sag mitigating devices for micro-level customer in distribution system," *IET Renewable Power Gener.*, vol. 13, no. 1, pp. 191–200, 2019.

[15] A. K. Goswami, C. P. Gupta, and G. K. Singh, "Cost-benefit analysis of voltage sag mitigation methods in cement plants," in *Proc. 16th Int. Conf. Harmon. Qual. Power*, May 2014, pp. 866–870.

[16] J. Y. Chan and J. V. Milanović, "Assessment of the economic value of voltage sag mitigation devices to sensitive industrial plants," *IEEE Trans. Power Del.*, vol. 30, no. 6, pp. 2374–2382, Dec. 2015.

[17] H. Liao, S. Abdelrahman, and J. V. Milanović, "Zonal mitigation of power quality using FACTS devices for provision of differentiated quality of electricity supply in networks with renewable generation," *IEEE Trans. Power Del.*, vol. 32, no. 4, pp. 1975–1985, Aug. 2017.

[18] Y. Ma, X. Xiao, and Y. Wang, "Investment strategy and multi-objective optimization scheme for premium power under the background of the opening of electric retail side," *Energies*, vol. 11, no. 8, Aug. 2018, Art. no. 2135.

[19] C. S. Chang and S. W. Yang, "Tabu search application for optimal multiobjective planning of dynamic voltage restorer," in *Proc. IEEE Power Eng. Soc. Winter Meeting. Conf.*, 2000, pp. 2751–2756.

[20] V. K. Awaar, P. Jugge, and T. Kalyani S, "Mitigation of voltage sag and power quality improvement with an optimum designed dynamic voltage restorer," in *Proc. IEEE Int. Conf. Power Electron. Drives Energy Syst.*, 2016, pp. 1–5.

[21] M. Farhadi-Kangarlu, E. Babaei, and F. Blaabjerg, "A comprehensive review of dynamic voltage restorers," *Int. J. Elect. Power*, vol. 92, pp. 136–155, Nov. 2017.

[22] J. D. Barros and J. F. Silva, "Multilevel optimal predictive dynamic voltage restorer," *IEEE Trans. Ind. Electron.*, vol. 57, no. 8, pp. 2747–2760, Oct. 2010.

[23] M. R. Khalghani, M. A. Shamsi-Nejad, and M. H. Khooban, "Dynamic voltage restorer control using bi-objective optimisation to improve power quality's indices," *IET Sci. Meas. Technol.*, vol. 8, no. 4, pp. 203–213, Jul. 2014.

[24] D. Patel, A. K. Goswami, and S. K. Singh, "Voltage sag mitigation in an Indian distribution system using dynamic voltage restorer," *Int. J. Elect. Power Energy Syst.*, vol. 71, pp. 231–241, 2015.

[25] B. Q. Khanh, "On optimally positioning a multiple of dynamic voltage restorers in distribution system for global voltage sag mitigation," in *Proc. IEEE Asia Power Energy Eng. Conf.*, 2019, pp. 137–143.

[26] F. Pilo, G. Pisano, and G. G. Soma, "Considering voltage dips mitigation in distribution network planning," in *Proc. IEEE Lausanne Power Tech*, 2007, pp. 1528–1533.

[27] S. Majumder, A. P. Agalgaonkar, S. A. Khaparde, P. P. Ciufo, S. Perera, and S. V. Kulkarni, "Allocation of common-pool resources in an unmonitored open system," *IEEE Trans. Power Syst.*, vol. 34, no. 5, pp. 3912–3920, Sep. 2019.

[28] L. Geest, "Three essays on defending common-pool resources," Ph.D. dissertation, Dept. Resour. Econ., UMass Amherst, Amherst, MA, USA, 2017. [Online]. Available: https://scholarworks.umass.edu/dissertations_2/1083

[29] J. Pitt, B. Dídac, B. Aikaterini, and P. Patricio, "Collective intelligence and algorithmic governance of socio-technical systems," in *Social Collective Intelligence: Combining the Powers of Humans and Machines to Build a Smarter Society*, D. Miorandi, V. Maltese, M. Rovatsos, A. Nijholt, and J. Stewart Eds., Cham, Switzerland: Springer, Aug. 2014, pp. 31–50.

[30] J. Pitt and J. Schaumeier, "Provision and appropriation of common-pool resources without full disclosure," in *Int. Conf. Princ. Pract. Multi-Agent Syst.*, Berlin, Germany: Springer, 2012, pp. 199–213.

[31] G. Bravo and B. Marelli, "Irrigation systems as common-pool resources. examples from northern Italy," *J. Alpine Res.- Revue de Géographie Alpine*, pp. 15–26, 2008. [Online]. Available: https://journals.openedition.org/rga/536

[32] D. B. Gillies, "Solutions to general non-zero-sum games," in *Contributions to the Theory of Games*, H. W. Kuhn and A. W. Tucker, Eds., Princeton, NJ, USA: Princeton Univ. Press, 1959, pp. 47–85.

[33] D. K. Foley, "Lindahl's solution and the core of an economy with public goods," *Econometrica*, vol. 38, no. 1, pp. 66–72, Jan. 1970.

[34] D. Schmeidler, "The nucleolus of a characteristic function game," *SIAM J. Appl. Math*, vol. 17, no. 6, pp. 1163–1170, Nov. 1969.

[35] L. S. Shapley, "Cores of convex games," *Int. J. Game Theory*, vol. 1, no. 1, pp. 11–26, Dec. 1971.

[36] GAMS - The Solver Manuals, *GAMS Release* 27, *GAMS Develop. Corporation*, Washington, DC, USA, 2019, Accessed: Jul. 2019. [Online]. Available: https://www.gams.com/latest/docs/S_MAIN.html

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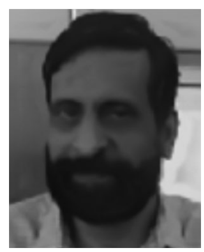

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