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Optimal Voltage Sag Mitigation Solution Provision using Customers Approximate Marginal Willingness-to-Pay Function

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***Abstract*—The requisite cost-benefit analysis for the deployment of voltage sag mitigation solutions often requires an explicit mathematical expression of the customers' willingness-to-pay estimates. The willingness-to-pay function, which is also equal to the benefit received by customers through the prevention of process failures, is a function of the probability density function of sag occurrence, customer's conditional process failure probability, and the process interruption cost. In this paper, through a detailed mathematical analysis, it has been shown that the customer's marginal willingness to pay function is linearly decreasing. The conditional probability theory is used for calculating the total probability, which is primarily utilized for calculating benefits received from the prevention of process failures. The derived linear willingness to pay function of the customer has been utilized to obtain optimal voltage sag mitigation solution provision for a typical customer. The symbiotic behavior resulting from common mitigation solution provision for multiple customers is also presented.**

***Index Terms*—Cost-benefit analysis, Probability theory, Voltage sags**

I. Introduction

AMONG various power quality (PQ) related concerns, the economic impact of short interruptions induced by discrete, stochastic, but frequent temporary faults in the overhead distribution network are very high. However, unlike short interruptions, the propagation of voltage sags induced by temporary fault events influences a large customer base. Moreover, both short interruptions and voltage sags are inevitable as a part of the protection devices coordination mechanism [1].

Higher frequency of voltage sags and enormous cost implication on commercial and industrial loads [2] necessitate sag performance improvement of the distribution network. Although with the recent establishment of various PQ-related regulations in different parts of the world, the voltage sag occurrence frequency can be significantly reduced; one cannot entirely eliminate its occurrence without significant economic investment, which ultimately will be passed onto the customers. Therefore, the cost-benefit analysis of voltage sag mitigation devices gains significance.

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A multitude of mitigation devices, such as SVCs (static VAR compensators), STATCOMs, etc., exist for voltage sag improvement. Optimal allocation of these mitigation devices for minimization of the weighted sum of the number of sags [3], network-wide overall utility maximization [4], aggregated performance improvement requirement [5], premium power investment strategy with disappointment-rejoicing psychological perceptions of sensitive customers [6], providing differentiated quality to various customers [7] have been discussed in the literature. Furthermore, once the mitigation device is in place, its benefit can be enjoyed by a group of customers. The cost-benefit analysis for sag mitigation of a typical industrial plant with specific process failure characteristics [8] is presented in [9]. Furthermore, the correlation among incentive provision from mitigation devices and plant sizes for various mitigation solutions is derived in [10].

Based on the existing literature, given a voltage sag event, the expected loss incurred with the customer's process failures depends on both the probability of observation of a certain residual voltage (the minimum voltage observed during a sag event) at the customer's premises and the conditional failure probability of the customer's processes. In this regard, methodologies for the stochastic assessment of observed residual voltage (such as Monte-Carlo analysis [11], [12]) are well established, and a detailed literature survey on the risk of process failures is also available in [13]. While one can readily calculate the impact of sags on customers based on Monte Carlo approaches or field measurements as discussed in the literature, a simple mathematical expression that can help one to carry out a primitive cost-benefit analysis for an optimal voltage sag mitigation solution provision without looking into the detailed plant model do not exist. In this paper, through an approximated mathematical analysis, we aim to estimate the customer's willingness-to-pay function for the voltage sag mitigation (which is also equal to the benefit received by preventing customers' process failures), which will supplement detailed mathematical analysis presented in [14], [15].

Our contributions in this paper are twofold: (i) Derivation of an analytical expression of customer's benefit from the mitigation solution provision. To achieve the same, the calculation of the probability density function (PDF) of the post-fault residual voltage and the yearly expected number of fault events are briefly discussed first. Subsequently, based on a simplistic conditional process failure probability function, the expected losses incurred with and without mitigation solution provision for a typical customer are calculated to extract the benefit from the solution provided. (ii) The proposed analytical benefit function is utilized to derive the optimal voltage sag mitigation solution provision considering DVRs and is presented through

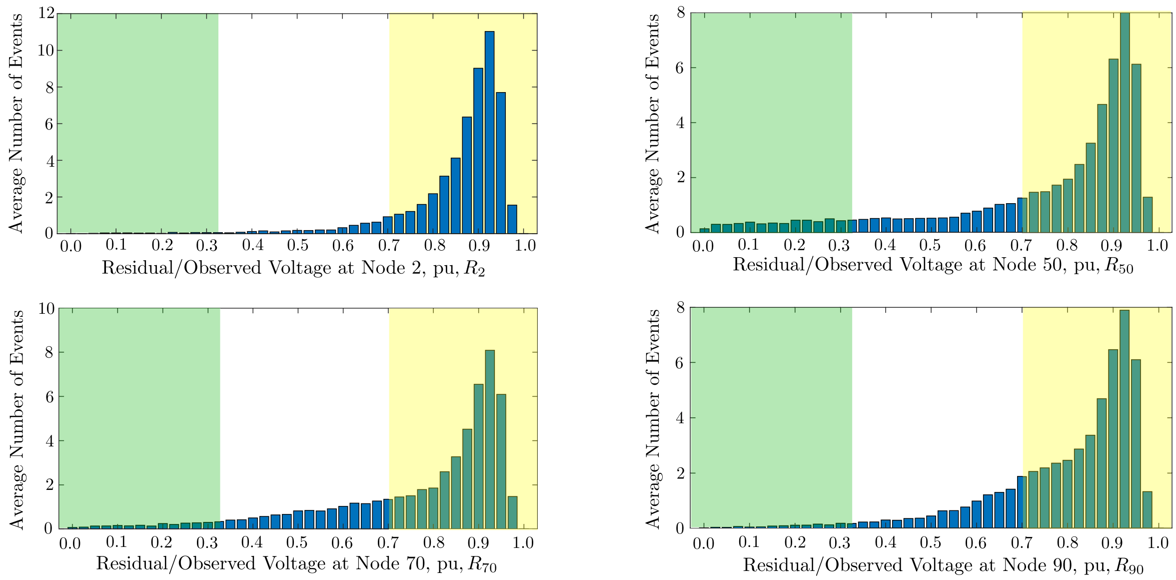

Fig. 1: Pictorial depiction of average number of sag events observed by various customers within an year.

a simple example.

II. Customers Willingness-to-Pay Estimate

A. Calculating the PDF of dropped Voltage

An IEEE 98-bus radial distribution network has been considered for calculating the sagged voltage PDF. Three radial IEEE 34-bus networks [16] are connected to the upstream network, where buses 1 and 2 of all the three networks are common. Here, distribution networks observe a source impedance of 0.0096+j2.1128 Ω connected between buses 1 and 2.

It has been considered that the customers experience voltage sags due to faults within the local network. The sags can also originate from the upstream network. In this regard, a fault rate of 55.92 faults/100 circuit-km/year is considered at each of the branches for fault analysis. The line length of one kilometer is considered to calculate the fault rate of the branches. Additionally, a truncated normally distributed [17] fault impedance, with the mean value and the standard deviation of 1 Ω and 5 Ω respectively, is also accounted for in the analysis. The state duration sampling approach [18] has been considered to obtain the fault statistics.

The discussion on the methodology of deriving residual voltage statistics is beyond the scope of this paper, and the existing literature [11], [19] can be referred to in this regard. Typical statistics of the residual voltage corresponding to the sag events observed by typical observers at various locations within a year occurrence frequency at nodes 2, 50, 70, and 90 (arbitrarily chosen) for the considered network are shown in Fig. 1. Similar characteristics can also be observed in Fig. 8 of [11] or Figs. 2, 3 and 4 of [12] or Fig. 7 of [19], which validates the observed sag statistics. As indicated in the existing literature, while the observed peaks in the residual voltage statistics is bound to change depending on radiality, length, fault rate of the distribution lines, fault rate of the upstream network, and relative location of the customer within the network; after observing the highlighted part of Fig. 1 it is imminent that the overall characteristics remain the same (residual voltage tend to concentrate near 0.9 *pu* with a very high frequency as shown in the yellow highlighted region, and gradually decreasing in frequency with decreasing residual voltage as given in the white highlighted region).

If the residual voltage is $R_i = (1.0 - V_i)$, then an additional voltage of V_i *pu* needs to be injected during the fault event to ensure normal operating condition at bus 'i'. It is well known that a dropped voltage of less than 0.05 *pu* is usually considered to be within normal voltage variation. Therefore, for calculating the cumulative probability, one can ignore the residual voltage of more than 0.95 *pu* under reasonable accuracy. Consequently, the cumulative probability of dropped voltage within 0.05 – 1.00 *pu* needs to remain non-negative and close to unity.

A rational approximant has been considered for estimating the PDF of the dropped voltage, as given by:

$$f(V_i) = \frac{a - bV_i}{V_i}; \quad \text{with } a \geq 0 \tag{1}$$

Exact methodology for the derivation of PDF from the obtained dropped voltage statistics is not given here for brevity. As shown in Fig. 2, the PDF of the typical dropped voltage during voltage sags, or the voltage to be injected satisfies the indicated analytical expression while the coefficient of determination of the fit is 95%.

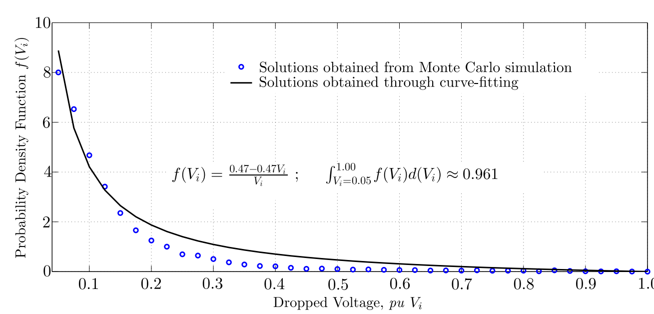

Fig. 2: The dropped voltage frequency distribution at an arbitrarily chosen bus.

As shown in Appendix A, for the PDF (1), if $b \geq 0$, then, $a \geq b$. Additionally, if one requires that the cumulative probability to remain within bound for 0.05 – 1.00 *pu*, then $a \leq 1/(-ln(0.05) - 1.00 + 0.05)$ (≈ 0.49). If, $b < 0$, then a will be limited to $1/(-ln(0.05))$ (≈ 0.33) and b will be limited to $-1/(1.00 - 0.05)$ (≈ −1.05), for the cumulative probability to remain bounded within 0.05 – 1.00 *pu*. The values of a and b obtained in Fig. 2 satisfies these indicated criterion. Additionally, one can also obtain annual average sag occurrence frequency, N, at a given bus, from the dataset presented in Figs. 2, 3, and 4 of [12], or various other literature. For the network under consideration, the annual average event occurrence frequency is 54.

B. Obtaining the Probability of Process Failure

The customer's process (or load) tripping probability is a function of the observed dropped voltage. Other parameters, such as the sag duration and the point-on-wave, are ignored in this work for simplicity. Although linearly increasing process failure probability curves are considered in [20], the failure characteristics also depend on the type of sensitive equipment present within the customer premises. To alleviate this complexity, a simplified customer's conditional process failure probability, $\mathbb{P}\left(F=1|V_i\right)$, where the probability of the process failure, $F = 1$, is defined to be linearly increasing with dropped voltage, is considered in this paper for calculating the benefits of mitigation solution provision.

$$\mathbb{P}\left(F=1|V_i\right) = V_i \tag{2}$$

C. Calculating the Willingness-to-Pay

1) Calculating the Total Cost without Mitigation Solution Provision: Calculation of the risk of process failure or the risk of loss is depicted in Fig. 3.

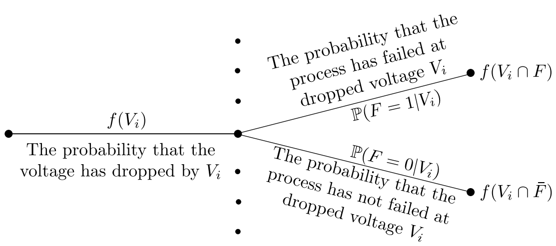

Fig. 3: Calculation of failure probability.

The total probability of loss can be given by:

$$\begin{aligned}\mathbb{P}\left(F=1\right) &= \int_{0.05}^{1.00} \mathbb{P}\left(F=1|V_i\right) f\left(V_i\right) dV_i \\ &= \int_{0.05}^{1.00} V_i \left(\frac{a-bV_i}{V_i}\right) dV_i = \int_{0.05}^{1.00} \left(a-bV_i\right) dV_i \\ &\leq \int_{0.00}^{1.00} \left(a-bV_i\right) dV_i \end{aligned}\tag{3}$$

The upper limit has been used to obtain the overall loss probability without mitigation solution provision. As shown in Appendix B, the total probability of failure is less than unity for all possible cases (b is ≥ 0 and < 0), and therefore, the considered upper bound is mathematically feasible (from the probability point of view).

If an expected N_i number of independent events occur in a typical year (obtained from Monte-Carlo simulation) and given that the customer i incurs a cost of C_i per event with probability $\mathbb{P}\left(F=1\right)$ (obtained from (3)), the yearly expected total expense perceived by the customer (T_i) will be:

$$T_i = C_i N_i \int_{0.00}^{1.00} \left(a-bV_i\right) dV_i \tag{4}$$

In the absence of a mitigation solution with an expected life of 'n' number of years (following the year of installation), the life-cycle cost (LCC) incurred, $LCC_i^{n,d}$, (based on the discount rate, d) can be given by:

$$\begin{aligned}LCC_i^{n,d} &= \frac{(1+d)^{n+1}-1}{d(1+d)^n} C_i N_i \int_{0.00}^{1.00} \left(a-bV_i\right) dV_i \\ &= K_i \int_{0.00}^{1.00} \left(a-bV_i\right) dV_i\end{aligned}\tag{5}$$

It is imminent that K_i is a constant for a typical customer.

2) Calculating the Total Cost with Mitigation Solution Provision: Suppose a mitigation solution with the capability of injecting a voltage up to V is in place. In that case, the equipment failure probability, with the dropped voltage being V_i $(\leq V)$, will be equal to zero. However, if the dropped voltage has increased beyond V, owing to the mitigation solution provided, the customer at the bus 'i' will only observe a residual voltage of $(1.00 - V_i + V)$. Contrarily, the dropped voltage PDF remains unchanged. Therefore, the conditional process failure probability will be needed to be modified based on observed residual voltage. In this regard, the total probability of process failure will be given by:

$$\begin{aligned}&\left(\int_{0.00}^{V} 0 \cdot \frac{a-bV_i}{V_i} dV_i\right) + \left(\int_{V}^{1.00} \left(V_i - V\right) \frac{a-bV_i}{V_i} dV_i\right) \\ &= \left(\int_{V}^{1.00} \left(a-bV_i\right) dV_i\right) - \left(V \int_{V}^{1.00} \frac{a-bV_i}{V_i} dV_i\right) \\ &= \left(\int_{V}^{1.00} \left(a-bV_i\right) dV_i\right) - V\left\{a \ln\left(\frac{1.00}{V}\right) - b(1.00-V)\right\} \\ &\qquad \leq \left(\int_{V}^{1.00} \left(a-bV_i\right) dV_i\right) - (a-b)V(1.00-V)\end{aligned}\tag{6}$$

In (6), if $V_i \leq V$, the process failure probability is zero, but the PDF of the sag occurrence is $\frac{a-bV_i}{V_i}$. Owing to mitigation solution provision of V, if the sagged voltage is V_i $(\geq V)$, the customer will only observe a voltage reduction of $V_i - V$, which will also be the probability of customers process failure (see, (2)). However, the PDF of the sag occurrence would still be $\frac{a-bV_i}{V_i}$. As shown in Appendix C, the upper limit ensures that the PDF in (6) remains mathematically feasible for all possible cases (total probability is always less than unity). We utilize the upper limit for calculating the process failure probability with the mitigation solution provision for simplicity.

The LCC considering the mitigation solution provision with the capability injecting a voltage up to V, $\overline{LCC}_i^{n,d}(V)$, can be similarly calculated using the simplified process failure probability, utilizing the methodology given in Section II.C.1.

3) Incentive Received from Mitigation Solution Provision: Therefore, the savings received due to installation of the mitigation solution up to voltage V will be calculated by subtracting $\overline{LCC}_i^{n,d}(V)$ from $LCC_i^{n,d}$, and one gets:

$$K_i \left\{\left(\int_{0.00}^{V} \left(a-bV_i\right) dV_i\right) + \left(a-b\right) V \left(1.00 - V\right)\right\} \tag{7}$$

The constant part of the total benefit received is zero. The marginal benefit received from the installation of the mitigation solution, which can also be the customers' marginal willingness-to-pay, can be obtained by differentiating savings received (given in (7)) with respect to the maximum injectable voltage, V. It can be given by,

$$W_i(V) = K_i\,(2a-b)\,(1-V) = \alpha_i\,(1-V) \qquad (8)$$

Here, α_i is constant for a given customer. In the current context, $\alpha_i \geq 0$ (note that $(2a-b)$ is always non-negative). The probability theory also suggests that the marginal benefit function is zero beyond the interval $[0,1]$. Such a linearly decreasing marginal benefit function has been assumed in [14] (with slight generalization) to calculate the optimal voltage sag mitigation solution provision.

D. Discussion on the Derived Linearly Decreasing Benefit Function

The magnitude of the slope and the y-intercept of the marginal willingness-to-pay function, α_i, is a function of several parameters. Here, the factor $(2a-b)$ is dependent on various network-related parameters, such as network topology, fault rates, and the location of the customers within the network. The factor K_i is obtained by multiplying several parameters, such as the annual frequency of occurrence of sag events, the expense per failure events, and the aggregated present value factor. The characteristics of the marginal/total benefit function are shown in Fig. 4.

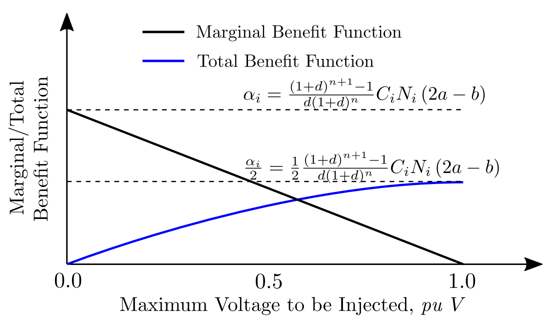

Fig. 4: Pictorial representation of simplistic willingness-to-pay function.

Suppose multiple customers are located on the same bus. In that case, although the PDF of dropped voltage and the annual sag occurrence frequency can be similar, the expense per failure events and associated aggregated present value factor will determine the slope and the y-intercept. If multiple customers choose to collaborate in the mitigation solution provision, in the linearly decreasing marginal benefit function, the x-intercept will remain constant at 1.00 *pu*. However, the y-intercept of the aggregated marginal benefit will be a linear sum of the maximum marginal willingness-to-pay (total willingness to pay will be sum of individual willingness to pay, and α_i of individual customers will be algebraically added). The benefit function also signifies that a sensitive customer with higher process failure costs located in a network with higher annual network-wide fault occurrence will likely be willing to invest more in the mitigation solution. Correspondingly, the slope of the marginal benefit function will be higher.

III. Optimal Mitigation Solution Provision with DVRs

Therefore, various network-related parameters are needed by the customers to be able to derive their willingness-to-pay function using Monte Carlo simulation. The customers may also deduce these parameters through multi-year power-quality measurements. Also, the literature indicates the existence of the economy of scale in the sag mitigation solution provision, such as DVRs, and the details can be found in [4]. As discussed in [14], the existence of such an economy of scale can be utilized by the customers to install a common mitigation solution. Generic average total cost (investment + net present value of operation and maintenance cost) characteristics of mitigation solution/desired revenue requirement, as reported in the literature (after removal of the quadratic component of the average cost curve with negligible weight), can be given by,

$$A^P(S_i) = c - dS_i \implies A^P(V) = cI_i - d(I_i)^2 V \qquad (9)$$

Here, S_i and I_i are power rating and peak load current demands of individual customers. Also, c and d are positive constants, and are different for different mitigation solutions. We consider DVR as a mitigation solution in this work; and given the peak load current demands, the average cost function, (9), is suitably converted to be a function of residual voltage V alone. Also, it is trivial that (9) is defined within $[0, \frac{c}{2dI_i}]$, since total cost is decreasing for $\left(\frac{c}{2dI_i}, \infty\right)$ and is negative for $(-\infty, 0)$.

Consequently, the total profit as a function of mitigation solution provided, V, will be: $(\alpha_i - cI_i)\,V - \left(\frac{1}{2}\alpha_i - d(I_i)^2\right) V^2$. The profit function is quadratic, the stationary point is $\frac{\alpha_i - cI_i}{\alpha_i - 2d(I_i)^2}$ and existence of maxima and minima will be driven by $\frac{1}{2}\alpha_i - d(I_i)^2$. The stationary point is a maxima, if $\alpha_i > 2d(I_i)^2$, it is a saddle point if $\alpha_i = 2d(I_i)^2$, otherwise it is a minima. Therefore, it is imminent that depending upon c, d, α_i and I_i, four conditions for calculating the optimal voltage rating, V^*, of mitigation devices exists. Also, it is important to note that the total profit is zero at $V = 0$.

Condition A. $\alpha_i < cI_i$ & $c \geq 2dI_i$: Three cases are possible. (i) $2d(I_i)^2 < \alpha_i < cI_i$, where the stationary point is at $V^* < 0$ and it is a maxima. Therefore total profit with $V \geq 0$ is always negative and mitigation solution will never be provided. (ii) $\alpha_i = 2d(I_i)^2 \leq cI_i$, where the profit function is linearly decreasing and always negative for $V \geq 0$. And, (iii) $\alpha_i < 2d(I_i)^2 \leq cI_i$, where the stationary point is a minima with $V^* \geq 1$. Even if $V^* = 1$, it is easy to capture that the total profit is negative. We define, $V^* = 0$ for this condition.

Condition B. $\alpha_i \geq cI_i$ & $c \geq 2dI_i$: Two cases are possible here. (i) If, $\alpha_i > 2d(I_i)^2$, then, $V^* = \frac{\alpha_i - cI_i}{\alpha_i - 2d(I_i)^2}$, and it is a maxima. Also, V^* is in $[0,1]$, and the total profit is non-negative. (ii) If, $\alpha_i = 2d(I_i)^2$, note that both $\alpha_i - cI_i$ and $\alpha_i - 2d(I_i)^2$, would tend to zero, and if we take the limit, we get $V^* = 1$. Without loss of generality we define $V^* = \frac{\alpha_i - cI_i}{\alpha_i - 2d(I_i)^2}$ for this condition.

Condition C. $\alpha_i \leq cI_i$ & $c < 2dI_i$: Here $\alpha_i \leq cI_i < 2d(I_i)^2$ signifies, V^* is in $[0,1]$ and it is a minima. Therefore, mitigation solution will be provided, if the profit is nonnegative at $V = \frac{c}{2dI_i}$ (beyond this the marginal cost function is undefined), that is, if, $\alpha_i \geq \frac{cI_i}{2 - \frac{c}{2dI_i}}$.

Condition D. $\alpha_i > cI_i$ & $c < 2dI_i$: Three cases are possible. (i) If, $cI_i < \alpha_i < 2d(I_i)^2$, the stationary point is at $V \leq 0$ and it is a minima. (ii) $cI_i < \alpha_i = 2d(I_i)^2$, Profit function is linearly increasing. And, (iii) $cI_i < 2d(I_i)^2 < \alpha_i$, here the stationary point is at $V > 1$, and this point is a maxima. In all the three cases, if the total profit is non-negative at $\frac{c}{2dI_i}$, that is, if, $\alpha_i \geq \frac{cI_i}{2-\frac{c}{2dI_i}}$, and we define, $V^* = \frac{c}{2dI_i}$ for this condition.

Conditions C and D can be aggregated together as: if, $\alpha_i \geq \frac{cI_i}{2-\frac{c}{2dI_i}}$ and $c < 2dI_i$, then, $V^* = \frac{c}{2dI_i}$; else, $V^* = 0$. All these four conditions are presented in a condensed form for a more generic case in Definition 2 of [15].

As discussed earlier, here, α_i is a function of the cost of the individual process failure, most of which is incurred due to sensitive process. The optimal rating of mitigation solution provision (V^*) will be:

$$V^* = \left\{0, \frac{c}{2dI_i}, \frac{\alpha_i - cI_i}{\alpha_i - 2d(I_i)^2}\right\} \tag{10}$$

Therefore, if both sensitive and non-sensitive components are considered for the determination of peak load demand, the voltage rating of the mitigation solution will be improperly sized, which is not desirable. If the sensitive and non-sensitive components can be isolated, we need to install mitigation devices for each of the sensitive components separately. Alternatively, multiple sensitive components can be aggregated together to install a common mitigation solution. Numerous customers can also participate in this venture and are presented through an example in the next section.

While the common solution provision can be cost-effective, such a joint venture suffers from one major disadvantage: the free-riding of any of the customers. This is because the voltage improvement for the benefited customers are non-excludable and is independent of their contribution status. The cost distribution to avoid such a free-riding (with $c \geq 2dI_i$) has been discussed in [14], and the multiple cluster formations to avoid free-riding has been discussed in [15].

IV. Example with DVRs as a Mitigation Solution

The sag-related information can be obtained from the sample modified distribution network considered in Section II. The values of both a and b at the bus under consideration are 0.47. Given the fault rate at the indicated bus, the customer's annual number of sag events, N_i, is 54.

The basic cost related parameters for a typical industrial customer used in this case study is primarily taken from [4], and is given as follows:

(i) The cost of sudden interruption of \$ 21,516 is considered as maximum sag cost (C_i), and will be encountered, if the sag leads to complete process failure.

(ii) The unit investment cost of DVR (with removal of small quadratic term) is given as,

$$T^P(S_i) = 729.96(-0.3225S_i + 127.38) \text{ \$/MVAr} \tag{11}$$

(iii) The annual operational and maintenance cost is assumed to be 15% of the investment cost. Therefore, the average total cost will be:

$$A^P(S_i) = \left(1 + \sum_{t=1}^{n} \frac{0.15}{(1+r)^t}\right) T^P(S_i) \text{ \$/MVAr} \tag{12}$$

(iv) The lifetime of the DVR (n) is assumed to be 20 years, and the investment will be recovered at a discount factor (d) of 12 %.

Customers of various peak load demands have been considered for calculating the rating of the DVR as a mitigation device, and successively, the characteristics of the overall benefit received from mitigation device installation has been shown in Fig. 5. It can be seen that with increasing peak load demand, for the given dataset, both the optimal voltage rating of the DVR and, consequently, the benefit received decreases. While the benefit function is independent of customers' load demand rating, the average cost function is (ratings of underlying devices are required to be suitably increased) increasing. Let us consider that the parameters that dictate Condition B exists (see, Section III). It is imminent that with increasing peak load demand (I_i), with the condition remaining similar, V^* will be decreasing. With increasing I_i three additional conditions (Conditions A, C, and D) may prevail. With Condition A, V^* reduces to zero. With Condition C, V^* can either be zero, or, $\frac{c}{2dI_i}$. It is notable that since $c < 2dI_i$, and $\alpha_i \neq 2dI_i$, $\frac{\alpha_i - cI_i}{\alpha_i - 2d(I_i)^2} \geq \frac{c}{2dI_i}$. Thus V^* is decreasing. This is also true if condition D prevails. If conditions A and C prevail initially, with increasing I_i, it will continue to remain there. If condition D prevails initially, with increasing I_i, it may jump to either 0 or continue to remain in $\frac{c}{2dI_i}$. Therefore, with increasing I_i, V^* is in general decreasing, as seen in Fig. 5. It can be further noted that if the cost of sudden interruption increases, the voltage sag mitigation solution provision is expected to rise.

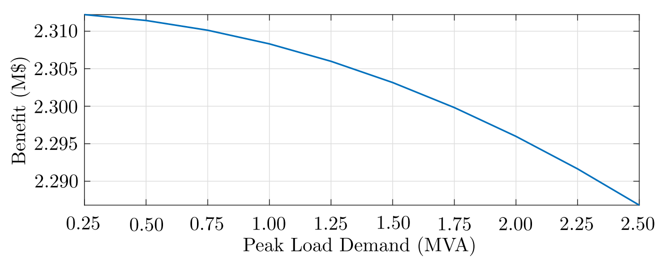

Fig. 5: Optimal benefit received by a customer with increasing peak demand.

From this figure, the customer's lifetime benefit with 0.75 MVA peak load demand will be \$2.3100M. Suppose multiple customers located on the same bus, and each with a rating of 0.75 MVA and identical cost of sudden interruption, are willing to contribute to a common solution. Calculation shows that in this case, the benefit received by each customer will be increased to \$2.3102M (total peak load demand increased along with total appropriation). Such an increased benefit received will be coupled with decrements in the overall cost (see (11)) compared to the case where each customer installs their own mitigation device. Therefore, for the given case, it is beneficial for these customers to behave symbiotically. While, various other parametric analysis has also been carried out for the given example, but are not shown here for brevity.

V. Conclusion

In this paper, a rudimentary mathematical expression of the customer's marginal benefit function or the willingness-to-pay with respect to the maximum voltage injected from the mitigation devices is derived using elementary concepts of the probability theory. The linearly decreasing characteristic of the willingness-to-pay function is also described. Subsequently, the derived expression has been utilized to determine a mathematical expression of the optimal mitigation solution requirement (with DVRs as an example). The decreasing benefit with increasing peak-load demand is also shown but is expected to be a function of various customers' costs and network-related parameters under consideration. Increasing individual benefits and the decreasing cost with a common mitigation device installation is expected to incentivize the customers to behave symbiotically.

APPENDIX A

The probability density function (PDF), $\frac{a-bV_i}{V_i}$ (with $a \geq 0$), needs to be non-negative $\forall V_i \in [0.00, 1.00]$. This can happen in two cases, (i) $b \geq 0$, and (ii) $b < 0$. Furthermore, we choose ϵ because $\lim_{V_i \to 0} \frac{a-bV_i}{V_i} \to \infty$, and $\int_{\epsilon}^{1.00} \frac{a-bV_i}{V_i} \leq 1$. This assumption is justifiable considering $\pm 5\%$ voltage variation, or $\epsilon = 0.05$, to be within normal bound.

Case (i): If $b \geq 0$, then $a - bV_i \geq 0$, or $a/b \geq V_i$, $\forall\ V_i \in [0.00, 1.00]$. Which is true, if $a \geq b$. Additionally, $\int_{\epsilon}^{1.00} \frac{a-bV_i}{V_i} = -a \ln \epsilon - b(1.00 - \epsilon) \leq 1$. Now, because, $a/b \geq 1.00$, one can write $b = a - \gamma$ where, $\gamma \geq 0$. Substituting, we get: $a(-\ln \epsilon - 1.00 + \epsilon) + \gamma(1.00 - \epsilon) \leq 1$. Note that a, γ are required to be ≥ 0 in the current context. All these three inequalities suggests that the value of a will be limited to $1/(-\ln \epsilon - 1.00 + \epsilon)$. With ϵ = 0.05, a will be limited to $\approx$ 0.49.

Case (ii): If $b < 0$, then $a - bV_i \geq 0$, $\forall V_i \in [0.00, 1.00]$. Suppose, $a(-\ln \epsilon) + \gamma(1.00 - \epsilon) \leq 1$; where, $\gamma = -b > 0$. Note that a, γ, are required to be ≥ 0 in the current context. Therefore, $a \leq (1/-\ln \epsilon)$ and $b \geq -1/(1.00 - \epsilon)$ respectively. With ϵ = 0.05, the upper and lower limits of a and b are 0.33, and -1.05 respectively.

APPENDIX B

The existance of limits $0 \leq \int_{0.05}^{1.00} (a - bV_i)\, dV_i \leq \int_{0.00}^{1.00} (a - bV_i)\, dV_i \leq 1.0$ needs to be validated for both the cases indicated in Appendix A.

Case (i): Here, $b \geq 0$. Now, $\int_{0.05}^{1.00} (a - bV_i)\, dV_i = a(1.00 - 0.05) - b/2(1.00^2 - 0.05^2) = b(1.00 - 0.05)\,(a/b - 1/2(1.00 + 0.05))$. Since, $1/2(1.00 + 0.05) \leq 1$ and $a/b \geq 1$, therefore, $\int_{0.05}^{1.00} (a - bV_i)\, dV_i \geq 0$. Again, $\int_{0.00}^{1.00} (a - bV_i)\, dV_i = a(1.00 - 0.00) - b/2(1.00^2 - 0.00^2) \leq a = 1/(-\ln \epsilon - 1.00 + 0.05) \leq 1.0$. Furthermore, $a(1.00 - 0.00) - b/2(1.00^2 - 0.00^2) - a(1.00 - 0.05) - b/2(1.00^2 - 0.05^2) = 0.05b(a/b - 0.05/4) \geq 0$ ($b \geq 0$ and $a/b \geq 1.00$).

Case (ii): With $b < 0$, we consider $\int_{0.05}^{1.00} (a + kV_i)\, dV_i$, and $k = -b$. Here, the minimum of $a(1.00 - 0.05) + k/2(1.00^2 - 0.05^2)$ is 0. The maximum of $\int_{0.00}^{1.00} (a + kV_i)\, dV_i$ is $(-1/\ln 0.05) + 1/\left(2(1.00 - 0.05)\right)$, which is ≤ 1. Furthermore, $a(1.00 - 0.00) + k/2(1.00^2 - 0.00^2) - a(1.00 - 0.05) - k/2(1.00^2 - 0.05^2) = 0.05a + 0.05^2 k/2 \geq 0$.

APPENDIX C

Here, $\left(\int_{V}^{1.00} (a - bV_i)\, dV_i\right) - V\left\{a \ln\left(\frac{1.00}{V}\right) - b(1.00 - V)\right\} = a\,(1.00 - V - V\ln(1.00/V)) - b/2\,(1.00 - V)^2$. Again, $\left(\int_{V}^{1.00} (a - bV_i)\, dV_i\right) - (a - b)V(1.00 - V) = (a - b/2)\,(1.00 - V)^2$. It is trivial to note that $\ln(1.00/V) \geq (1.00 - V)$, $\forall V \in (0.00, 1.00]$, which lead us to $a\,(1.00 - V - V\ln(1.00/V)) - b/2\,(1.00 - V)^2 \leq (a - b/2)\,(1.00 - V)^2$. To prove the feasibility of the PDF, we need to prove that $a\,(1.00 - V - V\ln(1.00/V)) - b/2\,(1.00 - V)^2 \geq 0$ and $(a - b/2)\,(1.00 - V)^2 \leq 1.00$ for both the cases indicated in Appendix A.

Case (i): Here, $b \geq 0$ and $a/b \geq 1.00$. It is known that $2\,(1.00 - V - V\ln(1.00/V)) \geq (1.00 - V)^2$, $\forall V \in (0.00, 1.00]$. Which implies, $2a\,(1.00 - V - V\ln(1.00/V)) \geq a\,(1.00 - V)^2 \geq b\,(1.00 - V)^2$. This provides one with, $a\,(1.00 - V - V\ln(1.00/V)) - (b/2)\,(1.00 - V)^2 \geq 0$. Furthermore, in $V \in (0.00, 1.00]$, $(1.00 - V)^2 \leq 1.00$ and $(a - b/2) \leq a$. Therefore, $(a - b/2)\,(1.00 - V)^2 \leq a \leq 1.00$.

Case (ii): Again consider that $b < 0$ with $k = -b$. Now, both $(1.00 - V - V\ln(1.00/V))$ ($\forall V \in (0.00, 1.00]$) and $(1.00 - V)^2$ are ≥ 0. This provides us with $a\,(1.00 - V - V\ln(1.00/V)) + k/2\,(1.00 - V)^2 \geq 0$. Again, in $V \in (0.00, 1.00]$, $(1.00 - V)^2 \leq 1.00$ and $(a + k/2) \leq 1.00$. Therefore, $(a - b/2)\,(1.00 - V)^2 \leq 1$.

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