



# Distributed Optimization for the Resilient Power Grid

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# Enabling Resiliency

## Needs

- Number of increasing weather and cyber events
- Need proactive and corrective optimal control
- Need scalable solutions with increasing state and control variables

## Solution

- Distributed Optimization offers scalable and resilient control

## Challenges

- DO with discrete variable
- Nonlinear objective functions and constraints
- Deployment challenges

# What can we do about it?

## Optimization Algorithms

1

### Algorithms/ Tools

Distributed optimization algorithm considering continuous and discrete variables, faster convergence and accuracy to enable resilient control,



## Resilience Metric

2

### Metrics

Measure effectiveness of alternative solutions to enable resilience



## Testbed

3

### Testbed for Validation

Validate algorithms and tools for deployment



Key efforts are needed to solve the future grid problem with increasing extreme events



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**Metrics**  
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## Summary

 <p>Increasing adverse events and integration of distributed energy resources results in higher number of state and control variables and requires scalable and resilient solutions</p>	 <p>Metric is needed to compare alternative solutions to enable resilience</p>	 <p>Grid monitoring and control requires distributed solutions for scalability</p>	 <p>Distributed control and management is critical to enhance grid resiliency</p>	 <p>Supporting computing infrastructure need to be scalable and fault-tolerant for resilient DER-rich electric grid and need to validated with the testbed</p>
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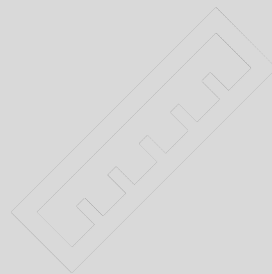


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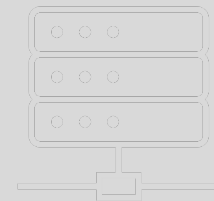


## Testbed

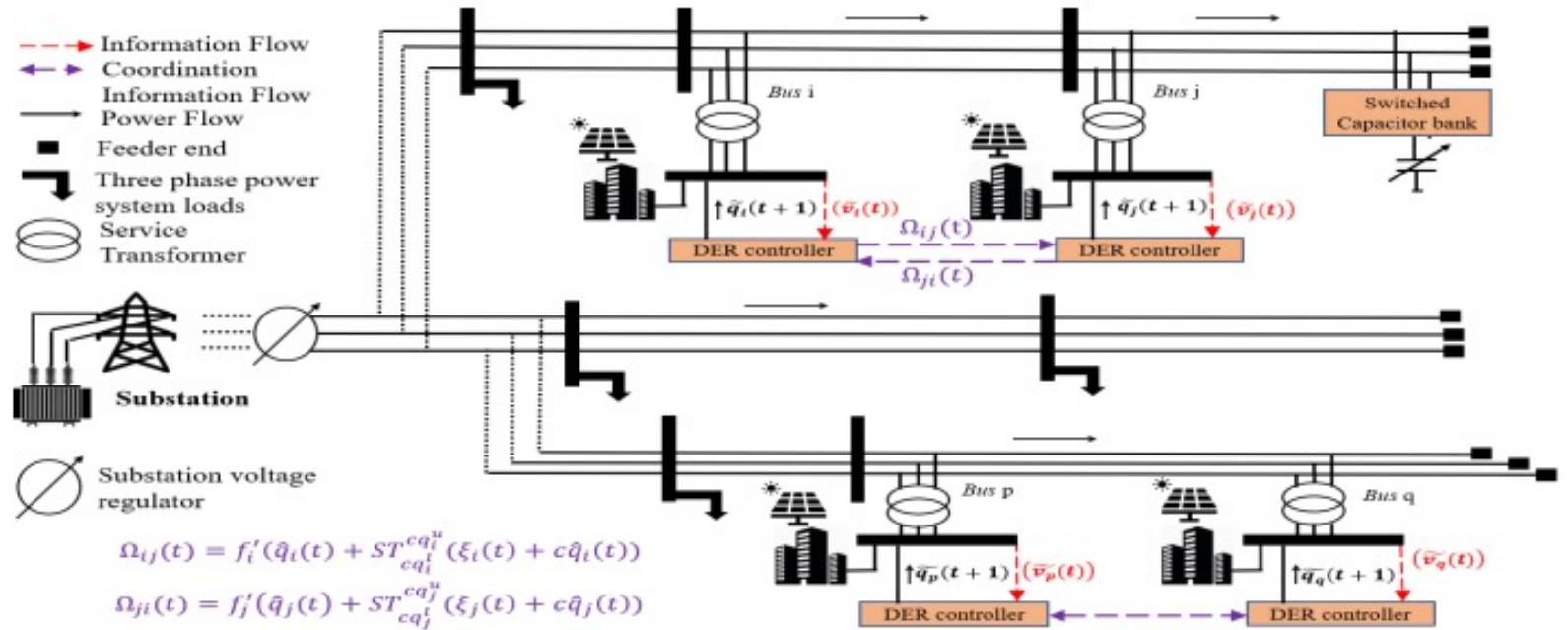
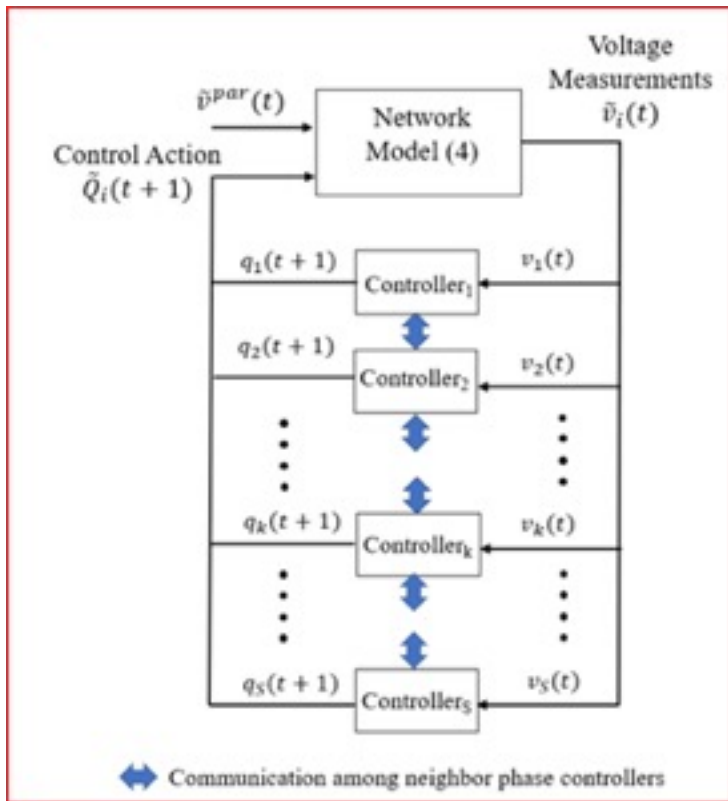
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# Distributed Volt-Var Control in Distribution System (OPT-DIST VC)



$$\min_{q_k} \triangleq \sum_{k=1}^S f_k(q_k) + \frac{d}{2} q^T \bar{Z}^q q$$

$$s.t. \quad \underline{v}_k \leq v_k \leq \bar{v}_k \quad \forall k \in \mathcal{S}$$

$$q_k \leq q_k \leq \bar{q}_k \quad \forall k \in \mathcal{S}$$

$$\tilde{v} = \bar{Z}^q \tilde{Q} + \tilde{v}^{par}$$

Each  $k^{\text{th}}$  controller performs the following steps

**Step 1 (Measuring):**  $v_k(t)$

**Step 2 (Calculating):**

$$\hat{q}_k(t+1) = \hat{q}_k(t) - \alpha \left\{ \bar{\lambda}_k(t) - \underline{\lambda}_k(t) + d\hat{q}_k(t) + \sum_{j \in \mathcal{N}_k} [Z^q]_{kj}^{-1} \left[ f'_j(\hat{q}_j(t)) + \text{ST}_{c\underline{q}_j}^{c\bar{q}_j}(\xi_j(t) + c_j(t)) \right] \right\}$$

$$\xi_k(t+1) = \xi_k(t) + \beta \frac{\text{ST}_{c\underline{q}_k}^{c\bar{q}_k}(\xi_k(t) + c_k(t)) - \xi_k}{c}$$

$$\bar{\lambda}_k(t+1) = [\bar{\lambda}_k(t) + \gamma(v_k(t) - \bar{v}_k)]^+$$

$$\underline{\lambda}_k(t+1) = [\underline{\lambda}_k(t) + \gamma(\underline{v}_k - v_k(t))]^+$$

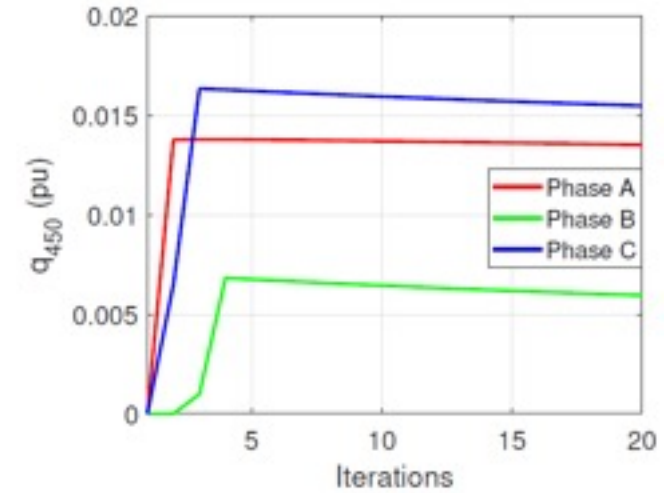
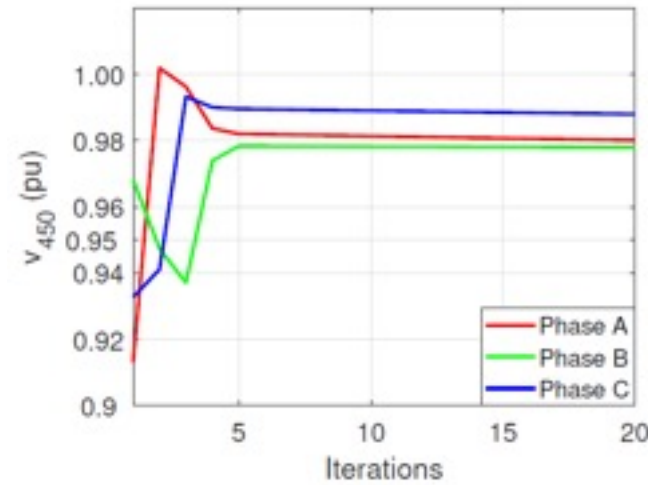
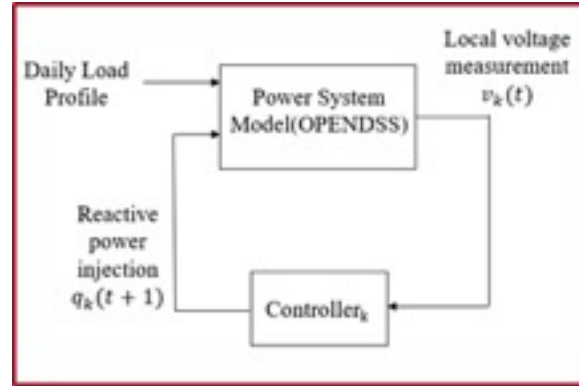
**Step 3 (Injecting Reactive Power):**  $q_k(t+1) = [\hat{q}_k(t+1)]_{q_k}^{\bar{q}_k}$

**Step 4 (Communicating):** Send values  $f'_k(\hat{q}_k(t+1)) + \text{ST}_{c\underline{q}_k}^{c\bar{q}_k}(\xi_k(t+1) + c_k(t+1))$  to neighbors  $\mathcal{N}_k$ .

Local variables

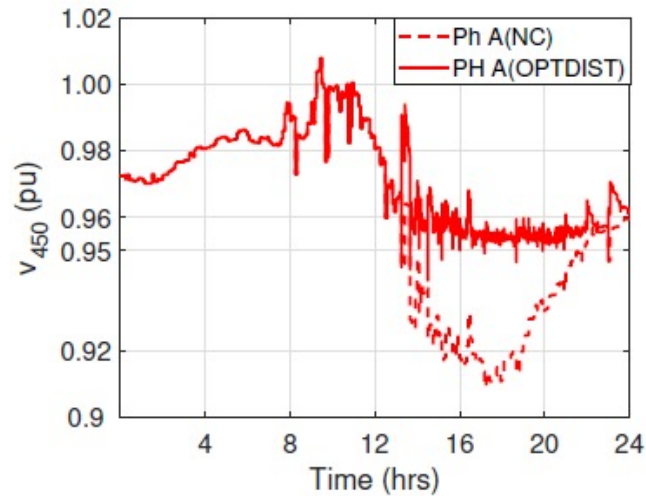
Block sparse matrix: Elements belonging to Only neighbor phase connections are non-zero, others are zero

Block sparsity enables this reactive power setpoint calculation to be distributed which means we only need  $\Omega$  values of neighbors

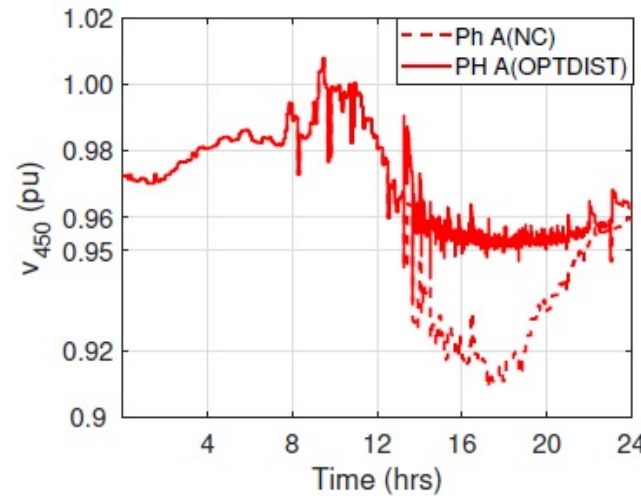


Voltage profile with distributed control

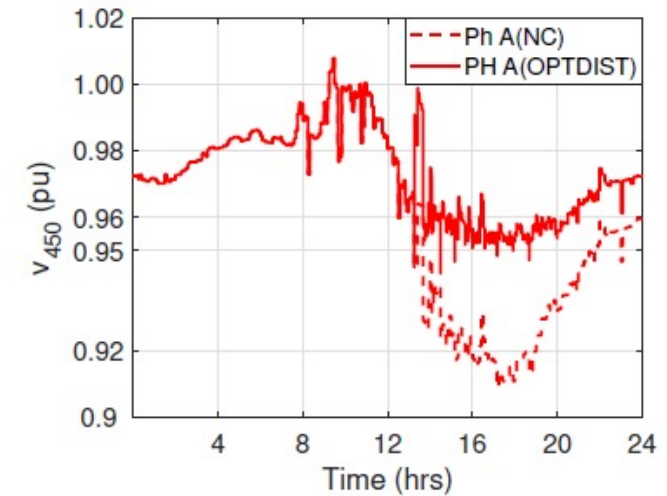
Reactive Power injection with distributed control



(a) Noisy measurements



(b) Communication Delays



(c) Modeling errors



# Distributed Optimization with Discrete Variables

- Legacy devices introducing Discrete Variables

- Switched Voltage Regulator

- On Load Tap Changing Transformers
- Switched Capacitor Banks



- During overvoltage condition due to surplus in PV generation, CBs can lower the voltage preventing PV active power curtailment
- During Under-voltage, SVR can improve voltage profile thus reducing system losses

- Switches

- Sectionalizer
- Recloser
- Tie Switches



At times of power outages, the smart co-ordination of switches can ensure network restoration in optimized way and improve self healing properties of grid.

**A coordinated operation of these devices leads to an efficient operation of the Power System**

# Distributed Optimization with Discrete Variables

## Challenges

Discrete Variables make the problem non-convex and non linear

Mixed Integer Non linear programming problem in a distributed way is an NP-hard problem

Implementing continuous relaxation to solve the optimization problem and rounding off later does not ensure convergence and might violate the constraints

In most cases, feedback based approaches are not suitable due to repetitive fluctuation

Evaluating the performance and choosing the best combination from a combinatorial analysis for the discrete variables is very difficult without a root subsystem in a completely distributed way

# Distributed Optimization with Discrete Variables

## Review of Key Approaches

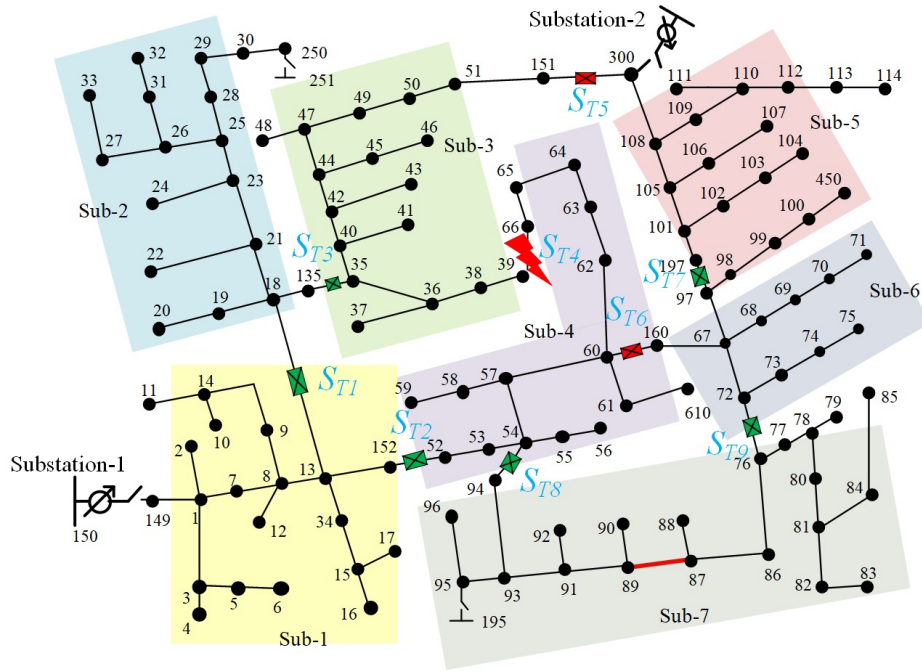
Ref.	Problem Spec	Obj. func. minimize	Discrete Algorithm	Distributed Algorithm	Boundary Variable	Communication requirements	Decision Variable	Comment
[1]	MIQP	Generation Cost & System Loss	Quadratic penalty term for non integer values	ADMM	Auxiliary variables representing the increments in real and imaginary part of voltages at boundary buses	Neighboring Areas	OLTC SCB	Guarantees convergence and optimality
[2]	MISOCP with cutting planes & Angle Relaxation	Active power curtailment Cost & system Loss	Branch & Bound	ADMM	Tie-line P and Q, primal and dual residual, boundary node voltage, objective function value of upstream and downstream region, SVR tap position	Neighboring areas	OLTC	No guarantee on convergence and optimality
[3]	MIQP	Generation Cost & System Loss	Ordinal Optimization	Dual Decomposition	Lagrange Multipliers and primal variables of boundary buses	Neighboring areas & root subsystems	OLTC SCB	Guarantees convergence to a good enough solution

[1] W. Lu, M. Liu, S. Lin and L. Li, "Incremental-Oriented ADMM for Distributed Optimal Power Flow With Discrete Variables in Distribution Networks," in *IEEE Transactions on Smart Grid*, vol. 10, no. 6, pp. 6320-6331, Nov. 2019

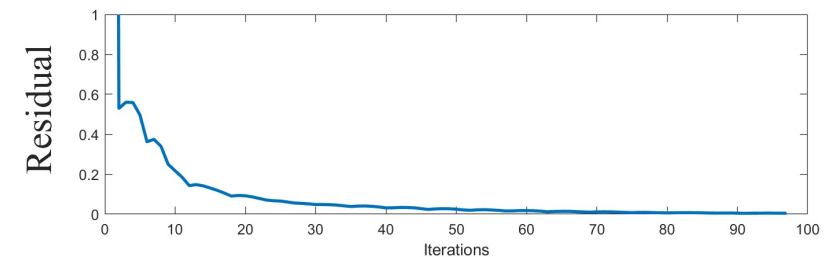
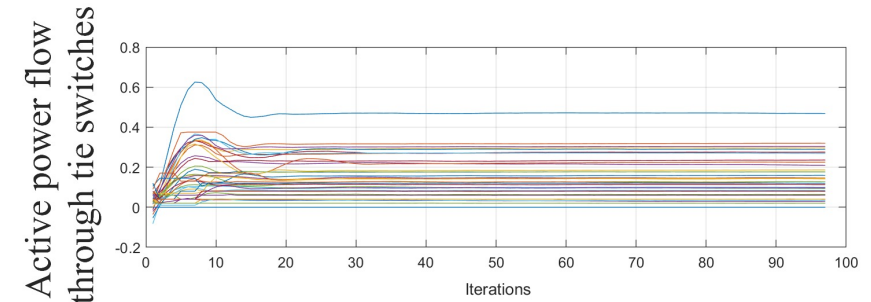
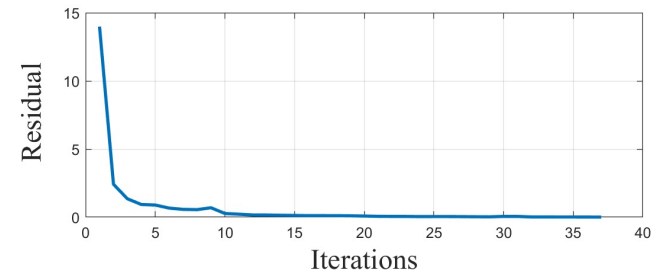
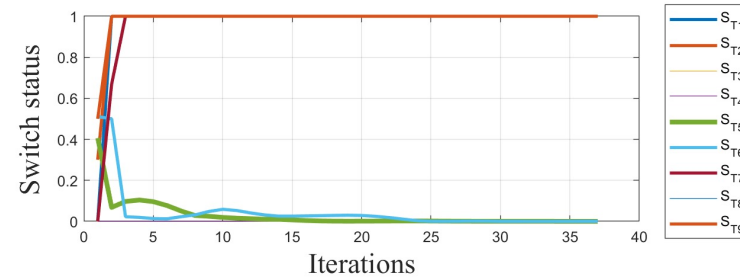
[2] Y. Liu, L. Guo, C. Lu, Y. Chai, S. Gao and B. Xu, "A Fully Distributed Voltage Optimization Method for Distribution Networks Considering Integer Constraints of Step Voltage Regulators," in *IEEE Access*, vol. 7, pp. 60055-60066, 2019

[3] C. Lin and S. Lin, "Distributed Optimal Power Flow With Discrete Control Variables of Large Distributed Power Systems," in *IEEE Transactions on Power Systems*, vol. 23, no. 3, pp. 1383-1392, Aug. 2008

# Distributed Approach for Optimal restoration: An Example



■ Tie-switch close    
 ■ Tie-switch open    
  Sectionalizing switch close    
  Sectionalizing switch open



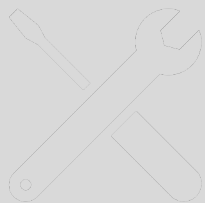


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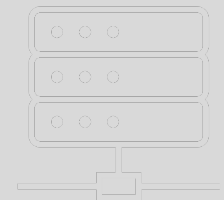


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Before Event

# A.W.R.

Inspired by CDC Public Health Emergency Preparedness and Response and Weighted Sum Model (WSM).

Measuring Resilience

## Anticipate

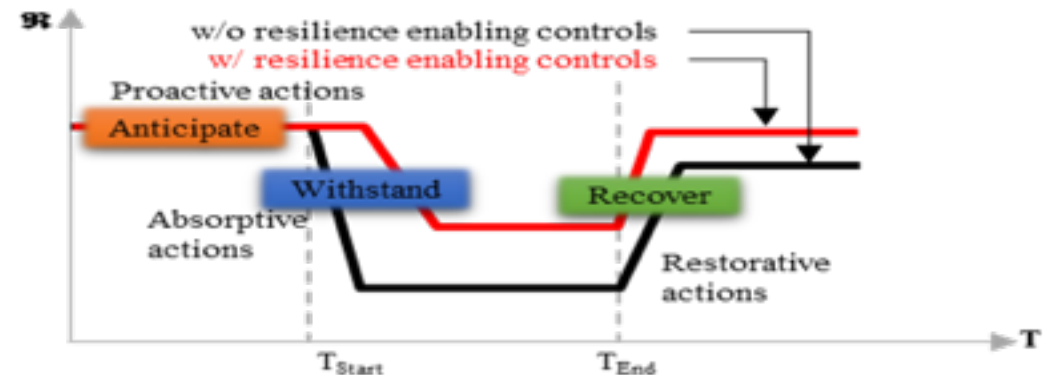
How well is the system prepared for the predicted impact of an incoming event?

## Withstand

How well can system continue to supply critical loads during event?

## Recover

How quickly can system recover from event and continue supply to critical loads? And at what cost?



During Event

# A.W.R.

Using system characteristics-based factors, graph theory, and Multi-Criteria Decision Making (MCDM): Analytical Hierarchical Process (AHP).

Measuring Resilience

## Anticipate

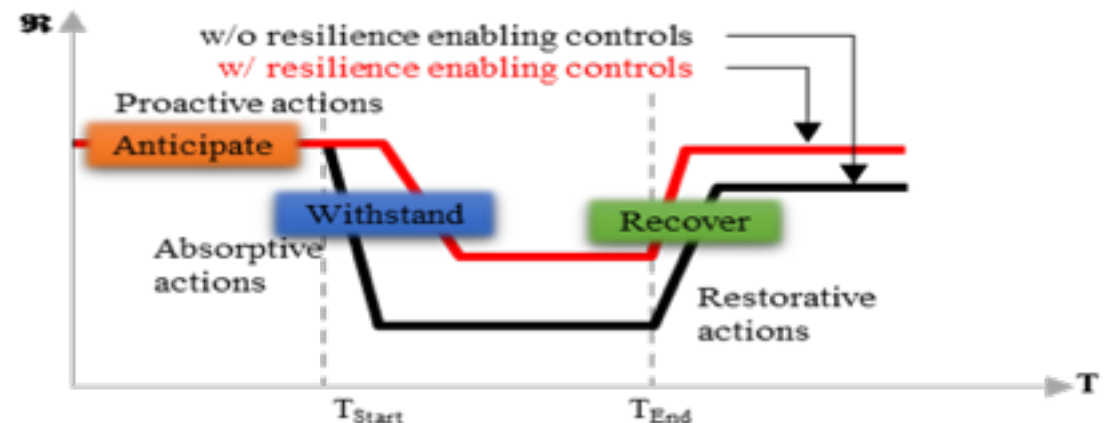
How well is the system prepared for the predicted impact of an incoming event?

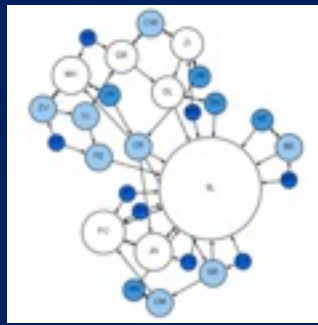
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After Event

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Measuring Resilience

## Anticipate

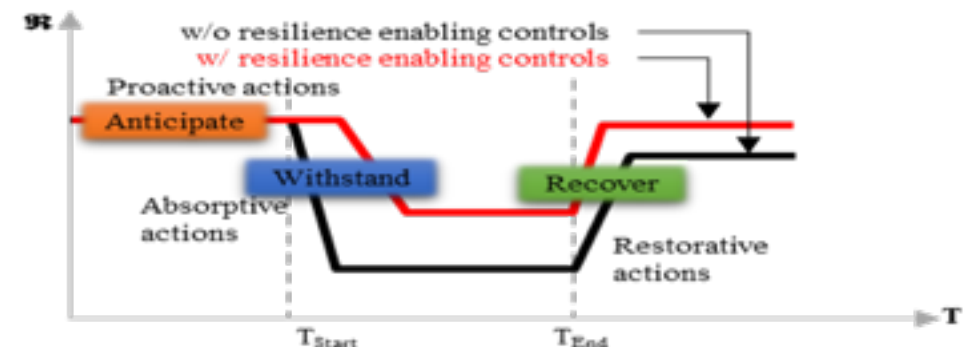
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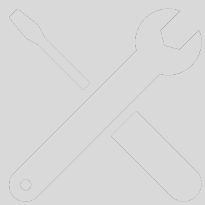


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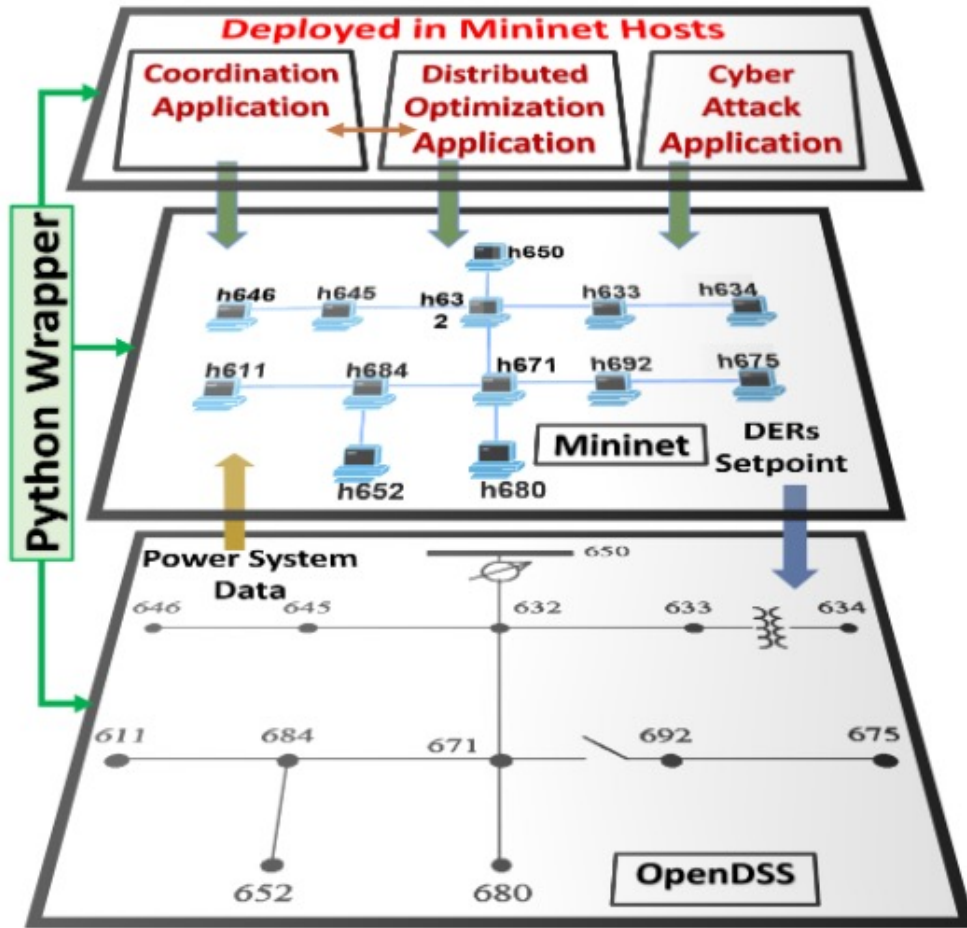
3

### Testbed for Validation

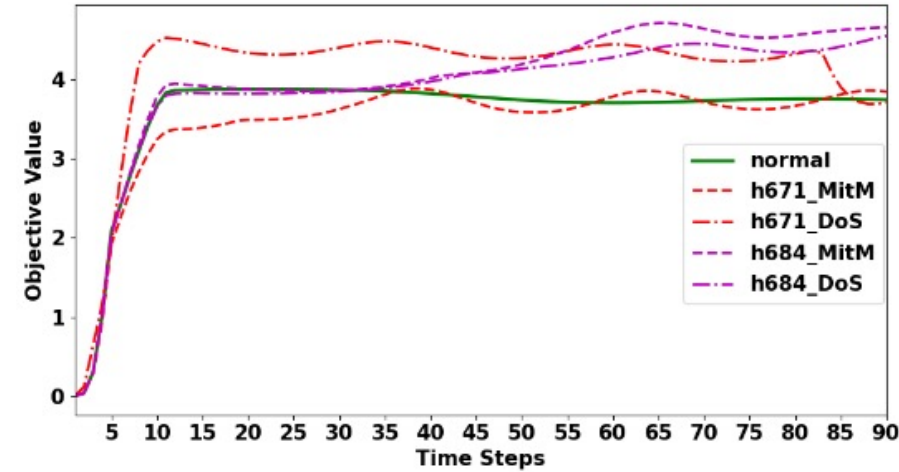
Validate algorithms and tools for deployment



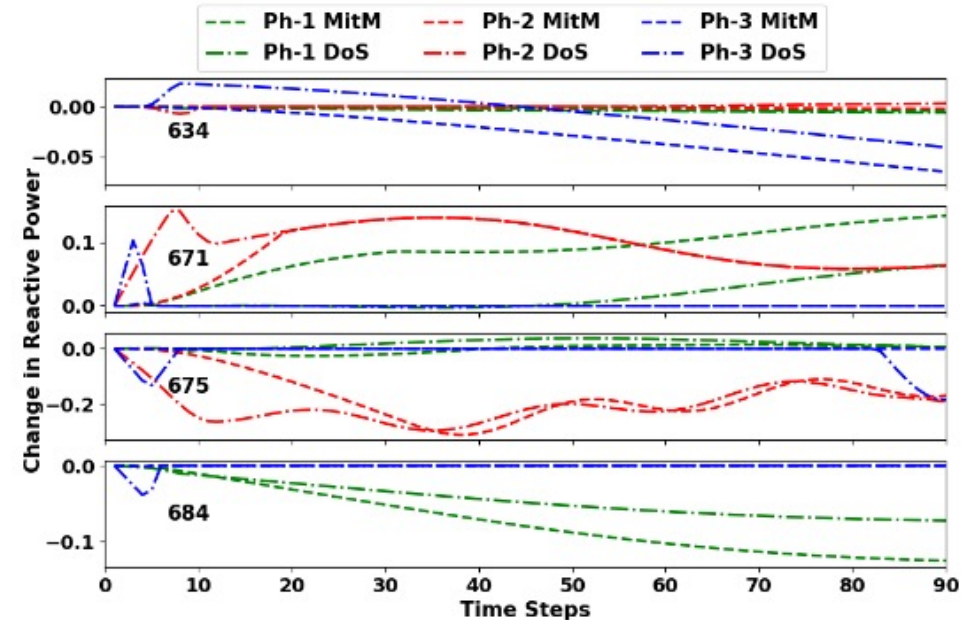
# Cyber-Physical Testbed: Distributed Volt Var Control in Distribution Systems



Test-bed architecture for distributed voltage control



Objective function during different scenarios



Change in reactive power injection in DER nodes

# Distributed Volt-Watt Control in Distribution Systems

OPTDIST-VWC: Each DER controller for a given node  $j$  ( $j \in \mathcal{N}$ ) follows four different steps at time  $t$ :

**Step 1 (Measurement):** Measure local voltages at all the available phases  $v_j(t)$ , and active power maximum power point  $p_j^{mpp}(t)$ .

**Step 2 (Calculating):** Calculate,  $\hat{p}_j(t+1)$ ,  $\xi_j(t+1)$ ,  $\bar{\lambda}_j(t+1)$ ,  $\underline{\lambda}_j(t+1)$ , using:

$$\hat{p}_j(t+1) = \hat{p}_j(t) - \alpha \{ (\bar{\lambda}_j(t) - \underline{\lambda}_j(t)) + \sum_{\forall i \in \mathcal{N}_j} [\bar{Z}^P]_{ji}^{-1} [f'_i(\hat{p}_i(t)) + \text{ST}_{-cp_j^{mpp}(t)}^0 (\xi_i(t) + c\hat{p}_i(t))] \}$$

(9a)

$$\xi_j(t+1) = \xi_j(t) + \beta \frac{\text{ST}_{-cp_i^{mpp}(t)}^0 (\xi_j(t) + c\hat{p}_j(t)) - \xi_j(t)}{c}$$

(9b)

$$\bar{\lambda}_j(t+1) = \bar{\lambda}_j(t) + \gamma [(v_j^{mcas}(t) - \bar{v}_j)]^+$$

(9c)

$$\underline{\lambda}_j(t+1) = \underline{\lambda}_j(t) + \gamma [(v_j - v_j^{mcas}(t))]^+$$

(9d)

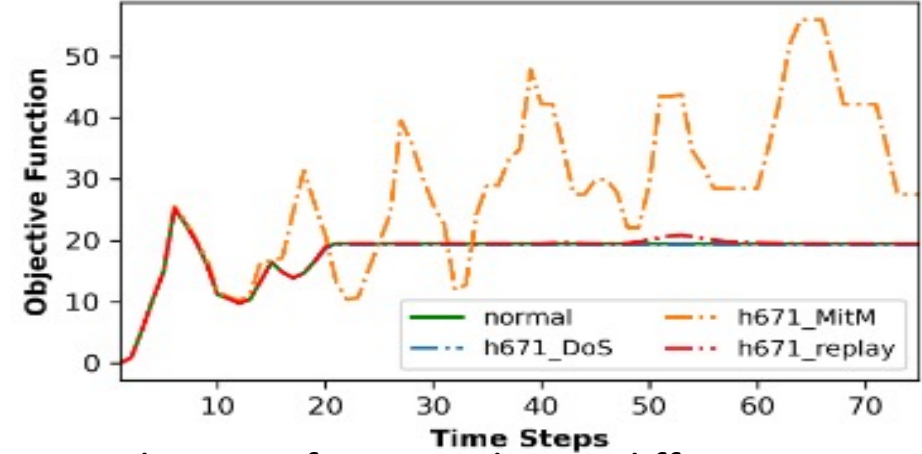
here,  $\mathcal{N}_j$  is the set of all neighbor nodes connected to node  $j$  ( $\forall j \in \mathcal{N}$ ).

**Step 3 (Active Power Set-Point Deployment):** Active power maximum power point is calculated again,  $p_j^{mpp}(t+1)$ . Active power injection set-point at time  $t+1$  is calculated as

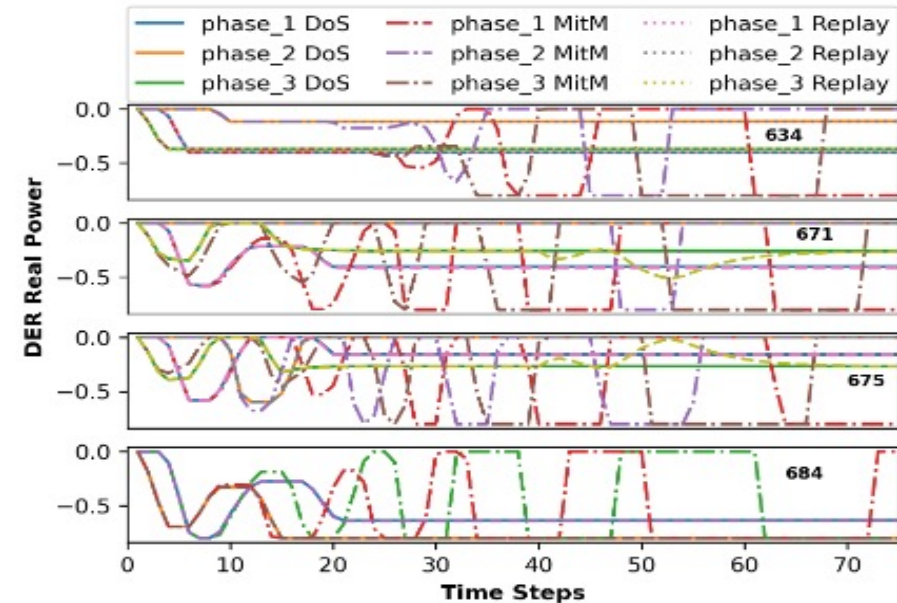
$$p_j^{inj}(t+1) = \left[ p_j^{mpp}(t) + [\hat{p}_j(t+1)]_{-p_j^{mpp}(t)}^0 \right]_{-p_j^{mpp}(t)}^{p_j^{mpp}(t+1)}$$

(10)

**Step 4 (Communication):** Values  $f'_j(\hat{p}_j(t+1)) + \text{ST}_{-cp_j^{mpp}(t+1)}^0 (\xi_j(t+1) + c\hat{p}_j(t+1))$  are communicated to neighboring DER nodes.  $\square$



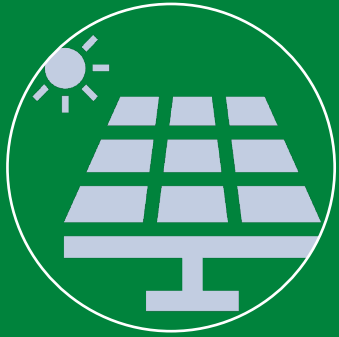
Objective function during different scenarios



Change in reactive power injection in DER nodes



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Metric is needed to compare alternative solutions to enable resilience



Grid monitoring and control requires distributed solutions for scalability



Distributed control and management is critical to enhance grid resiliency



Supporting computing infrastructure need to be scalable and fault-tolerant for resilient DER-rich electric grid and need to be validated with the testbed







Thanks to all the students and collaborators. We acknowledge support from the DOE UI-ASSIST and NSF CPS