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Revenue and ancillary benefit maximisation of multiple non-collocated wind power producers considering uncertainties

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Abstract: In this study, optimal scheduling of multiple non-colocated, price taker, independent wind power producers (WPPs) participating in forward day-ahead (DA) distribution electricity market is described; where, a WPP is comprised of multiple wind turbine generator (WTG) and battery storage device (BSD). Cost equivalent of reduction in network losses and improvement in voltage profile for non-colocated placement of WTG and BSD in Distribution Network (termed as ancillary benefit) is included in the objective function resulting in a scheduling strategy dependent upon location of WPP in the network. Objective function comprises of following sub-objectives: (i) maximize return from energy market, (ii) maximize benefit obtained from providing ancillary services, and (iii) minimize uncertainties in schedule by providing reserve from BSDs. Non-linear programming (NLP) technique is used for scheduling. Location of a WPP is varied to obtain a 'profit map'; which can be used as an 'offline-tool' to find out relative location of WTG and BSD for profit maximization. Proposed formulation is extended to participation of multiple WPP, where ancillary benefit is proportionally shared. Wind power forecast uncertainty leads to risk of not meeting the schedule and is probabilistically modeled in this work. Impact of reserve on DA energy schedule of is also studied.

Nomenclature

i	Elements of set of all buses, $i \in \{1, 2,\ldots, n_b\}$.
t	Elements of set of all time, $t \in \{1, 2,\ldots, n_t\}$.
w	Index for WTG.
b	Index for BSD.
d	Index for Load.
$\mathrm{mcp}(t)$	Day-Ahead market clearing price at time t, in \$/kWh.
$P_i^{(\cdot)}$	Active power output from WTG/BSD at bus i, in kW.
$\mathrm{Ct}_t^{(\cdot)}$	Total cost of generation from WTG/BSD, in \$.
$C_0^{(\cdot)}$	Fixed cost component of WTG/BSD, in \$/h.
$C_1^{(\cdot)}$	Variable cost component of WTG/BSD, in \$/kWh.
$V(i, t)$	Voltage magnitude of bus i at time t, in *pu*.
$L(t)$	Total loss at time t, in kW.
$Q_i^{(\cdot)}$	Reactive power output from WTG/BSD at bus i, in kVAr.
$\mathrm{SOC}(i, t)$	State of the charge (SOC) of BSD connected at bus i at time t.
$\mathrm{Cap}_i^{\mathrm{b}}$	Capacity of BSD connected at bus i, in kWh.
$\mathrm{SOC}^{\mathrm{begin}}$	SOC at the beginning of scheduling.
$\mathrm{SOC}^{\mathrm{end}}$	SOC at the end of scheduling.
$\mathrm{SOC}^{\mathrm{min}}$	Minimum possible SOC.
$\mathrm{SOC}^{\mathrm{max}}$	Maximum possible SOC.
$\mathrm{Rat}_i^{\mathrm{b}}$	Power rating of the BSD connected at bus i, in kW.
$P_{\min}$	Minimum possible active power dispatch from WPP to DSO, in kW.
$P_{\max}$	Maximum possible active power dispatch from WPP to DSO, in kW.
$S_{\mathrm{conv}}^{(\cdot)}$	WTG/BSD Converter rating, in kVA.
$V(i, t)$	Node voltage of the bus i at time t, in pu.
$\delta(i, t)$	Node voltage angle of the bus i at time t, in degree.
$\mathrm{Ptot}(t)_k$	Total power dispatched by WPP, k at time t, in kW.
$x(i, t)$	Reserve power supplied by the BSD at bus i at time t, in kW.
$\mathrm{rmcp}(t)$	Day-ahead reserve market clearing price at time t, in \$/kWh.
$V^{\mathrm{R}}(i, t)$	Node voltage of the bus i at time t when reserve is supplied, in pu.
$\delta^{\mathrm{R}}(i, t)$	Node voltage angle of the bus i at time t when reserve is supplied, in degree.

1 Introduction

For maximising profit by selling power from non-dispatchable wind energy resources, storage devices are required to be scheduled. Moreover, an independent wind power producer (WPP) has to either buy reserve from day-ahead (DA) reserve market or generate the same from its own units to minimise inherent wind-power forecast error [1–4]. Also, if WPPs non-collocate (connect at different buses) their wind turbine generator (WTG) and battery storage device (BSD) in the network, performance of distribution network in terms of (i) network losses, (ii) voltage profile and (iii) overall energy efficiency [5–9] will improve. Generating power locally from these renewable based distributed generators (DG) will not only displace generation from conventional resources, but also reduce the total power purchased by distribution system operator (DSO) as well. Also, supplying reserve from BSDs will reduce WPPs risk of not meeting the schedule. Therefore, from WPPs point of view, DA scheduling and risk mitigation strategy of non-collocated WTG and BSD assumes significance.

Denholm and Sioshansi [10] have studied the benefits of collocating WTG and storage devices at 'wind centre' to maximise the transmission capacity utilisation. In this case, capacity factor of the transmission system is improved by transmission of power from 'wind centre' to 'load centre'. Korpaas *et al.* [11] have proposed a method for scheduling and operation of storage devices using dynamic programming method. Castronuovo and Lopes have worked on optimal daily joint operation strategy of wind-hydro system using Monte–Carlo technique [12] and a deterministic method [13] respectively. Angarita *et al.* [14] have presented a combined stochastic joint-optimisation of collocated wind-hydro system. Lagrangian relaxation technique based optimisation algorithm for hourly unit scheduling strategy, and its impact on locational marginal price, peak load-shaving and transmission congestion is discussed in [15].

Ahn *et al.* [16] have proposed a dispatch scheduling algorithm in which, fuel cost is minimised in grid connected condition while stable operation is maintained during islanded condition. Silva *et al.* [17] have presented a short-term (15-minute ahead) resource management methodology using DA and hour-ahead scheduling result along with short-term generation forecast with lesser uncertainty. Borghetti *et al.* [18] have presented an algorithm along with DA-scheduling algorithm for distributed resource management. Smart energy management system for DA scheduling of a micro-grid is proposed in [19] using a matrix real coded genetic algorithm. Scheduling problem in distribution network for vehicle to grid application is presented in [20]. Vehicles in this scheme can be connected into the network in a distributed fashion. Effect of high penetration of DGs on voltage swell and reverse power flow in the distribution network; and hence its effect on losses has been studied in [9]. Marvasti *et al.* [21] have proposed a mathematical model for maximising benefits of microgrid and Distribution Company in a decentralised way. Chen *et al.* [22] have described a method to find out optimal size of BESS for both grid-connected and islanded operation of Microgrid, based on renewable energy generation forecast. Operation scheduling strategy with an objective to achieve voltage control of each node in a distribution network while reducing losses based on previous day forecast information of photovoltaic generator output and load demand is described in [7]. Song *et al.* [8] have described the benefits of distributing DERs in the distribution network and corresponding efficiency improvement because of loss reduction and supporting network during abnormal operating condition.

Mazidi *et al.* [2] have proposed a two-stage stochastic objective function for operational cost minimisation with renewable energy generation; where, reserve requirement to minimise the forecast error is supplied by responsive load and dispatchable DG units. A two-stage stochastic optimisation problem to deal with uncertainty in renewable power generation and load demand with an objective to minimise cost function is solved in [3]. An expert energy management system has been proposed in [4], to find out an optimal operating point of WTG and other distributed energy resources in a Microgrid, for minimising total operation cost and net emission; while wind power forecast error has been reduced by energy storage devices.

In recent years, Dupka *et al.* [1] have investigated a DA joint optimal dispatch and risk mitigation strategy for collocated WTG and BSD system. Optimal dispatch schedule is obtained considering best forecast is available to the wind electric generator. A part of reserve is supplied from the storage devices owned by them to reduce risk from the forecast uncertainty in terms of expected energy not served. They have studied, (i) how storage devices helps to maximise the profit of WPP, and (ii) how BSDs will change the net amount of energy sold to reduce the amount of risk. In this paper, a methodology for profit maximisation scheduling of multiple WPPs with wind-battery hybrid system have been proposed. Profit maximisation objective function for each individual WPP consists of following three sub-objectives: (i) participation in DA forward electricity market for profit maximisation, (ii) selling benefits in terms of reduction in Losses and voltage deviation to the DSO as ancillary service, and (iii) providing reserve to minimise reduction in profit from forecast uncertainty. In multiple player scenario, following two approach for scheduling have been chosen: (i) schedule independently to maximise individual profit, and (ii) schedule cooperatively to attain combined maximum profit.

Uncertainty in wind generation forecast is higher compared with load and market clearing price (MCP) forecast [23]. Hence, reserve requirement will mostly depend upon uncertainty in wind generation forecast. ‘Self Reserve’ model has been considered in this paper; where, reserve of each WPP can be supplied from BSDs or bought from the DA reserve market. However, supplying reserve from self owned units will directly affect its DA energy schedule. Therefore, reserve and energy schedule from these storage devices are required to be joint-optimised [24].

Scheduling problem with multiple WPPs can be broken into following sub-problems:

A. (i) Finding optimal schedule of single player selling energy to the DSO at DA market clearing price (DAMCP).

(ii) With varying location of WTG and BSD in the network, a profit-map can be generated. This can be used as an ‘offline tool’ to show independence or dependence of schedule on location of WPP (WTG-BSD).

B. Extending the scheduling procedure for multiple players; where, players can schedule individually or cooperatively.

C. Joint-optimisation of energy and reserve for single player scenario.

2 Problem formulation

Operational profit of the WPP can be defined as profit obtained by selling scheduled power to the DSO. It has been observed from the existing literature that, wind generation pattern and MCP are not correlated [13]. Hence, current practice is to schedule active power to maximise the operational profit of WPP with the help of storage devices.

Fig. 1 shows, the framework in which proposed scheduling process has been tested. The radial balanced distribution network is connected at the sub-transmission level. Scheduling horizon has been divided into 24 hourly intervals. All WPPs sell their scheduled power to the DSO at DAMCP, where they are functioning as price taker. WPPs being independent producer, they do not have direct access to the network parameters, load forecast etc. Since, DSO wants to improve the performance of the network; they have to provide all these estimates to the WPPs for scheduling. All WPPs have their own wind generation forecast for scheduling. ‘only wind (OW)’ [13] generation strategy has been used by the WPP for scheduling the hybrid system, in which total energy dispatched is equal to total forecasted wind energy generation.

Because of high *R*/*X* ratio of distribution network, real power flow causes large voltage deviation at the far end buses. If a part of the total demand is met by the local generators, total power loss in the network will be significantly reduced. WTG and BSD can supply reactive power into the network through voltage source converter based devices. Losses are also reduced by supplying reactive power locally [25]. Hence for optimal operation, reactive power from BSD and WTG will also be scheduled. Thus, cost of non-collocating and providing reactive power from own unit is paid off in terms of improvement of system performance and revenue earned. Following assumptions have been considered in the problem formulation:

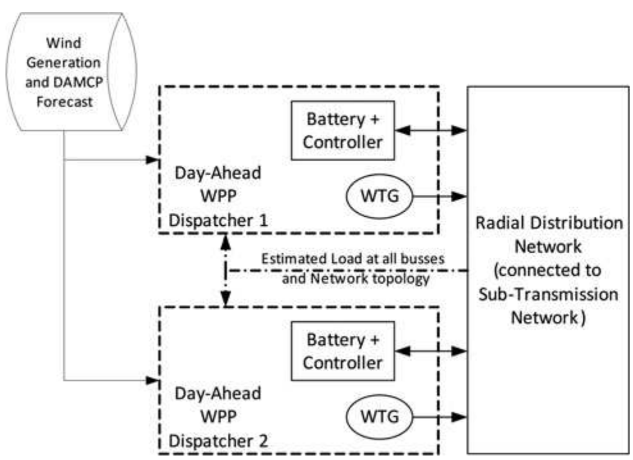

Fig. 1 *Framework for the proposed scheme*

(i) Size of all DGs are small compared to the total loading in the network. This is because, if DG size is relatively small compared with total loading, (a) line flow gets reduced, (b) voltage profile gets improved; but, nodal voltage limit will not hit.
(ii) Battery self-discharge rate is zero. Battery and converter efficiency is 100%. This leads to simplified model of state of the charge (SOC) of the battery.
(iii) DA forecast of wind generation, marginal prices and load are available to all WPPs.
(iv) DGs can be placed at any buses in the network without leading to network congestion. Advanced communication for operation and an agreement between DSO and WPPs for monitoring and control are in place, for non-collocated operation.
(v) The pdf of error in wind generation is having standard normal distribution with zero mean ($\mu = 0.000$) and standard deviation, σ of 12.973% [26].

All players submit their individual schedule to the DSO to distribute the ancillary benefit among them. In independent scheduling scenario, each WPP will schedule considering they are the only WPP present in the network. In this regard, individual players operational profit does not change, but ancillary benefit obtained is lesser than expected. In cooperative scheduling scenario, all WPPs are participating together in the market as a single WPP and submit their schedule to the DSO.

In the current problem, based on the above-mentioned five assumptions, following statements are hypothesised:

(i) If maximisation of operational profit is considered as the only objective, schedule is independent of location of WTG and BSD in the network.
(ii) If minimisation of losses and voltage deviation is included in addition with maximisation of operational profit, the schedule and therefore operational profit is dependent upon location of WTG and BSD.
(iii) Operational profit in the latter case is less than or equal to the earlier case, where only operational profit is considered as an objective function.

2.1 Single player energy scheduling problem

Revenue generation by selling scheduled power to the DSO at an hour t is $\mathrm{mcp}(t)$ times total power sold, and is depicted in the following equation. '+ *ve*' sign of generation from BSD represents that BSD is charging and vice versa.

$$\mathrm{Re}_t = \mathrm{mcp}(t) \cdot \sum_{\forall i} \{P^w(i,t) - P^b(i,t)\} \quad \forall t \tag{1}$$

Although, wind energy is available free of cost; empirical analysis shows that, cost of power production from the WTG is linearly dependent upon power output of the battery [1, 27]. Fixed cost is the installation cost of WTG and is independent of whether WTG is generating or not. Similar linear cost model has been used for the BSDs. Variable cost of BSD is independent of charging or discharging of the battery [1]. Cost of charging cycle of BSD is ignored in this paper. The cost of power production from WTG and from BSD are depicted in (2) and (3).

$$\mathrm{Ct}_t^b = \sum_{\forall i} C_0^b + C_1^b \cdot |P^b(i,t)| \quad \forall t \tag{2}$$

$$\mathrm{Ct}_t^w = \sum_{\forall i} C_0^w + C_1^w \cdot P^w(i,t) \quad \forall t \tag{3}$$

Voltage deviation index (VDI) and loss improvement index (LII) [28] has been used for quantifying the improvement in network performance, over the 'base case' scenario. 'Base case' is defined as the performance of the same network in which DGs are not placed. Equations depicting calculation of VDI and LII are shown below.

$$\mathrm{VDI} = \frac{\sum_{\forall t} \sum_{\forall i} [V(i,t) - V_{\mathrm{base}}(i,t)] \cdot \mathrm{mcp}(t)}{\max_t \mathrm{mcp}(t)} \tag{4}$$

$$\mathrm{LII} = \sum_{\forall t} \frac{\left(L_{\mathrm{base}}(t) - L(t)\right) \cdot \mathrm{mcp}(t)}{L_{\mathrm{base}}(t) \cdot \max_t \mathrm{mcp}(t)} \tag{5}$$

Both of these indices are weighted with forecasted DAMCP to obtain maximum benefit when MCP is high. $V_{\mathrm{base}}(i,t)$ and $L_{\mathrm{base}}(t)$ are 'base case' network voltage and total losses respectively.

Cost equivalent of ancillary benefits are calculated using welfare maximisation theory [29]. Cost equivalent of line loss improvement benefit is calculated by multiplying cost conversion factor for line loss improvement (C_l) by LII and Cost equivalent of voltage deviation reduction benefit is calculated by multiplying cost conversion factor of voltage improvement (C_v) by VDI. DGs reduce total network losses and thus total power purchase by the utilities. Hence, cost of providing loss improvement ancillary benefit has been calculated using average cost of reduction in power purchase.

The objective function of DA scheduling of WTG-BSD system for profit-oriented WPP is expressed as follows.

$$\max_{P^b, Q^b, Q^w, V, \delta} \underbrace{\sum_{\forall t} (\mathrm{Re}_t - \mathrm{Ct}_t^b - \mathrm{Ct}_t^w)}_{\text{Operational profit}} + \underbrace{C_\mathrm{v} \cdot \mathrm{VDI} + C_\mathrm{l} \cdot \mathrm{LII}}_{\text{Ancillary benefit}} \tag{6}$$

subject to,

$$P^w(i,t) - P^b(i,t) - P^d(i,t) = P_i(\boldsymbol{V}, \boldsymbol{\delta}) \quad \forall i, t \tag{7}$$

$$Q^w(i,t) + Q^b(i,t) - Q^d(i,t) = Q_i(\boldsymbol{V}, \boldsymbol{\delta}) \quad \forall i, t \tag{8}$$

$$\mathrm{SOC}(i,t) = \mathrm{SOC}(i,t-1) + \frac{P^b(i,t)}{\mathrm{Cap}_i^b} \quad \forall i, t \tag{9}$$

$$\mathrm{SOC}^{\min} \le \mathrm{SOC}(i,t) \le \mathrm{SOC}^{\max} \quad \forall i, t \tag{10}$$

$$\begin{aligned} \mathrm{SOC}(i,0) &= \mathrm{SOC}^{\mathrm{begin}} \\ &= \mathrm{SOC}^{\mathrm{end}} = \mathrm{SOC}(i, N_t) \quad \forall i, t \end{aligned} \tag{11}$$

$$-\mathrm{Rat}_i^b \le P^b(i,t) \le \mathrm{Rat}_i^b \quad \forall i, t \tag{12}$$

$$P^b(i,t)^2 + Q^b(i,t)^2 \le {S_{\mathrm{conv}}^b}^2 \quad \forall i, t \tag{13}$$

$$P^w(i,t)^2 + Q^w(i,t)^2 \le {S_{\mathrm{conv}}^w}^2 \quad \forall i, t \tag{14}$$

$$P_{\min} \le P^w(i,t) - P^b(i,t) \le P_{\max} \quad \forall i, t \tag{15}$$

$$0.80 \text{ pu} \le V(i,t) \le 1.05 \text{ pu} \quad \forall i, t \tag{16}$$

$$-90^\circ \le \delta(i,t) \le 90^\circ \quad \forall i, t \tag{17}$$

$$L(t) = f(\boldsymbol{V}, \boldsymbol{\delta}) \quad \forall t \tag{18}$$

$\boldsymbol{P}^b = \{P^b(i,t)\}$, $\boldsymbol{Q}^b = \{Q^b(i,t)\}$, $\boldsymbol{Q}^w = \{Q^w(i,t)\}$, $\boldsymbol{V} = \{V(i,t)\}$, $\boldsymbol{\delta} = \{\delta(i,t)\}$ represent the matrix of active power schedule from BSD, reactive power schedule from BSD and WTG, node voltage and angle respectively. Equations (7) and (8) are system power balance equations. Equation (9) is the model of SOC of the BSD; where, (10) depicts practical bound for reliable operation and maximum life of battery. SOC of the battery at the beginning ($t = 0$) and end ($t = N_t$) of scheduling horizon is depicted in (11) for 'OW' scenario. Power output from BSD is limited by power rating of the battery and is represented in (12). Equations (13) and (14) depicts WTG and BSD converter rating constraint. Equation (15) represents maximum and minimum possible dispatch limit from WPPs. Equations (16) and (17) represents node voltage and angle hard bound of the network when the network is providing reserve. Equation (18) represents, total line losses in the network is a function of node voltage, angle and network parameters. Given assumption (iv), line flow limits of all the lines in the network are not hit; and hence, constraints associated with line flow limits are not considered in this paper.

As seen from (6), optimal schedule is dependent upon location of BSD and WTG in the network. Moreover, given all the assumptions are satisfied; number of WTG and BSD acquired by WPP can vary to obtain its optimal schedule. When, number of BSD acquired by WPP is varied; WPP is interested about limit on net schedule of all BSD. Since, all the BSDs are connected to the network via converter; converter constraints and self energy constraints for each BSDs have to be satisfied.

Optimisation problem becomes highly non-convex when integer variables are introduced; and finding a global optimal solution to this kind of problem is very difficult. Hence, integer variables have been avoided by considering round trip efficiency and battery efficiency to be 100%. Possible changes in the problem formulation for round trip efficiency and battery efficiency to be other than 100% is described in Appendix 1.

2.2 Scheduling for multiple player and profit sharing

This sub-section describe the procedure of ancillary benefit sharing among participant WPPs. When multiple WPPs participate in the proposed market scenario, they can schedule in the following ways: (i) schedule independently assuming they are the only market participant (designated, method B1) or (ii) schedule cooperatively as a single player (designated, method B2). The flowchart depicting how benefit for providing ancillary services are distributed is shown in Fig. 2.

Several papers has been reported regarding loss allocation [30–33]. Proportional sharing of ancillary benefits has been considered in this paper, where profit of kth player is proportional to power generated by the corresponding unit. DSO distributes the ancillary benefits among the participants using (19) and (20).

$$\mathrm{VDI}_k = \sum_{\forall t} \frac{|\mathrm{Ptot}(t)_k|}{\sum_k |\mathrm{Ptot}(t)_k| + \Delta} \times \frac{\sum_{\forall i} [V(i,t) - V_{\mathrm{base}}(i,t)] \cdot \mathrm{mcp}(t)}{\max_t \mathrm{mcp}(t)} \quad (19)$$

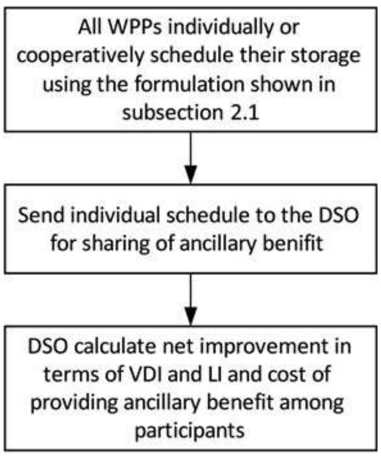

Fig. 2 *Flowchart showing profit sharing among WPPs*

$$\mathrm{LII}_k = \sum_{\forall t} \frac{|\mathrm{Ptot}(t)_k|}{\sum_k |\mathrm{Ptot}(t)_k| + \Delta} \times \frac{(L_{\mathrm{base}}(t) - L(t)) \cdot \mathrm{mcp}(t)}{L_{\mathrm{base}}(t) \cdot \max_t \mathrm{mcp}(t)} \quad (20)$$

In these equations, Δ is a small positive number to inhibit division by zero. Absolute value of power is considered for proportional scheduling; for net power may be zero, but individual schedule may not be zero. If players schedule individually and submit their schedule, received ancillary benefit will not be same as expected benefit.

2.3 Joint-scheduling of energy and reserve

DA joint-optimisation objective function of WTG-BSD system is expressed as follows.

$$\max_{P^b, X, Q^b, Q^w, V, \delta, V_R, \delta_R} \underbrace{\sum_{\forall t} (\mathrm{Re}_t - \mathrm{Ct}_t^b - \mathrm{Ct}_t^w)}_{\text{Operational profit}} + \underbrace{RP(X)}_{\text{Profit from 'self reserve' scheme}} + \underbrace{C_v \cdot \mathrm{VDI} + C_l \cdot \mathrm{LII}}_{\text{Ancillary benefit}} \quad (21)$$

$\boldsymbol{X} = \{x(i,t)\}$ is the matrix of reserve power supplied from BSD. Profit from providing reserve, $RP(\boldsymbol{X})$ is given by $\sum \gamma(t) \cdot x(i,t)^2 \cdot \mathrm{mcp}(t) - x(i,t) \cdot \mathrm{rmcp}(t)t$. Derivation of $RP(\boldsymbol{X})$ and definition of $\gamma(t)$ has been shown in Appendix 2. Following set of constraints along with constraints (7)–(18) are required to be satisfied for joint-optimisation.

$$\mathrm{SOC}^{\min} \le \mathrm{SOC}(i, t-1) + \frac{(P^b(i,t) \pm x(i,t))}{\mathrm{Cap}_i^b} \le \mathrm{SOC}^{\max} \quad \forall i, t \quad (22)$$

$$0 \le x(i,t) \le \sigma \mathrm{Cap}^w \quad \forall i, t \quad (23)$$

$$-\mathrm{Rat}_i^b \le P^b(i,t) \pm x(i,t) \le \mathrm{Rat}_i^b \quad \forall i, t \quad (24)$$

$$(|P^b(i,t)| + |x(i,t)|)^2 + Q^b(i,t)^2 \le {S^b_{\mathrm{conv}}}^2 \quad \forall i, t \quad (25)$$

$$P^w(i,t) - P^b(i,t) \pm x(i,t) - P^d(i,t) = P_i(\boldsymbol{V_R}, \boldsymbol{\delta_R}) \quad \forall i, t \quad (26)$$

$$Q^w(i,t) - Q^b(i,t) - Q^d(i,t) = Q_i(\boldsymbol{V_R}, \boldsymbol{\delta_R}) \quad \forall i, t \quad (27)$$

$$0.80\ \mathrm{pu} \le V_R(i,t) \le 1.05\ \mathrm{pu} \quad \forall i, t \quad (28)$$

$$-90^\circ \le \delta_R(i,t) \le 90^\circ \quad \forall i, t \quad (29)$$

$\boldsymbol{V_R} = \{V_R(i,t)\}$, $\boldsymbol{\delta_R} = \{\delta_R(i,t)\}$ represent the matrix of node voltage and angle respectively when reserve is supplied. Equation (22) depicts limits on SOC level of BSD with reserve. Equation (23) shows maximum reserve that can be allocated from BSD, which is dependent upon total capacity of WTG (Cap^w) and standard deviation of forecast uncertainty (σ). When, multiple BSDs are acquired by WPPs, sharing of reserve to be supplied by each of the BSDs are based on power rating of individual BSDs. Rating of BSD and WTG converters rating creates another set of constraints for reserve allocation, as shown in (24) and (25) respectively. Equations (26) and (27) are system power balance equation of the network; when the system is supplying reserve. Equations (28) and (29) represents node voltage and angle hard bound. Line flow limit constraint while supplying reserve is not considered. Three associated scenarios has been considered as a part of scenario reduction technique: (i) when supplied reserve is at its positive extreme, (ii) when supplied reserve is at its negative extreme, and (iii) reserve requirement is zero.

3 Case study

Proposed strategy has been tested on IEEE 11 kV, 33-bus radial distribution network [34]. Network topology with R and X in Ω and active power peak load (PLp) in kW, reactive power peak load (QLp) in kVAr is given in Appendix 3. For the case studies the location of the WPP (WTG-BSD) is varied. Various inputs for the optimisation problem, such as, forecasts of wind power [35], load [36], MCP [37] and reserve MCP forecasts [38] are depicted in Fig. 3.

Forecasted wind generation pattern is assumed to be same for all the generators. Wind power generation pattern is normalised with capacity of the wind generator. DA forecast of normalised load profile with peak load is considered to be same for all the buses. Power factor of the load remains constant at peak load power factor.

Capacity of each WTG is assumed to be 800 kW. Capacity factor of wind farm is 0.25 (annually approximated). Approximate generation in 24 hr is 800 kW × 24 h × 0.25 or 4800 kWh. In the worst case scenario, when MCP is high for 12 hours at a stretch, wind generation is low and when MCP is low for rest 12 hours, wind generation is high. Hence, in 1 day for single WPP case, BSD will undergo only one storage cycle. Then, storage requirement for this condition is 4800 kWh/2 = 2400 kWh. If SOC is allowed to vary within 0.2–0.8; then capacity of BSD will be 2400 kWh/0.6 = 4000 kWh. BSDs can be placed at one single location or distributed at multiple buses. In this paper, a single BSD of calculated capacity is considered for case study. The method for calculating size of BSD is general, and is applicable even if number of hours for which wind speed is high is changed.

Minimum generation from the WTG is 0 kW throughout the day. Under this condition power sold to DSO will be 0 kW; and thus it is minimum contract among DSO and WPP. Maximum generation of WTG is 800 kW throughout the day. In this case 2400 kW amount of energy can be stored for time shifting in 12 h to be sold in next 12 h according to MCP. In this case maximum power to be sold to DSO will be 800 kW + 200 kW = 1000 kW. Hence, maximum contract for WPP is 1000 kW. And thus, the rating of BSD is 200 kW.

Operation and maintenance (O&M) costs for WTG is \$0.016–0.020/kWh [27]. Fixed cost of wind turbine, \$12.8/day is taken from [1], and equally divided among 24 hours. Fixed and variable O&M cost of BSD (¢0.00001/kWh, ¢10.08/day) is also taken from [1]. Average DA marginal price is ¢3.5/kWh. Total load in the network is 50 MWh. Assuming, 15% losses in the network and 10% loss reduction (approx.) is possible in the distribution network, average possible loss reduction will be ¢720 kWh/day. Therefore, average benefit for providing loss reduction ancillary benefit is ¢3.5/kWh × 720kWh/day or \$25.00/day. Since, VDI and LII are network improvement attributes, and signifies average improvement in losses in the network, these attributes multiplied by average benefit will provide cost equivalent of ancillary benefits. Hence, cost conversion factor for line loss improvement (C_l) is calculated as \$25.00/day. Since, cost associated with the voltage improvement can't be measured directly; similar figures have been taken for cost conversion factor of voltage improvement (C_v).

The deterministic MINLP problem has been solved by standard branch and bound solver (SBB) of General Algebraic Modeling Software (GAMS) [39] using a Windows-based operating system with one Core 2 Quad processor clocking at 2.44 GHz and 8 GB of RAM. Maximum simulation time for various placements of WTG and BES is 50 s.

Following results are going to be discussed in this section:

(i) System and economic benefits of non-collocating BSD-WTG with single player scheduling.
(ii) Single player scheduling extended to participation of multiple WPPs and profit sharing between them.
(iii) Joint scheduling of energy and reserve for single player in DA market.

3.1 Schedule of BSD for single WPP

Scheduled power is calculated by summing up of the generation from BSD and WTG. Since, generation from the WTG is directly obtained from the forecast, SOC indirectly provides the schedule of WPP. Charging and discharging of BSD depends upon inputs of the objective function including location of WTG and BSD in the network. (bus_w, bus_b) represent the buses in which WTG and BSD respectively are connected. Changes in SOC profile with various WTG and BES location is shown in Fig. 4. SOC profile gets modified with change in location of WPP. This implies that WPPs maximises ancillary benefit reducing its operational profit.

When ancillary benefit is included into the objective function, operational profit will either remain same or reduce. Because, in this case, power sold into the energy market will either remain same or reduce for increasing ancillary benefit, justifying hypothesis (iii). Variation in SOC profile depends upon location of WPP (WTG-BSD) in the network. If WPP is located at node 1 and ancillary services are considered along with operational profit; net profit is found to be \$ 537.61. Net profit is independent of

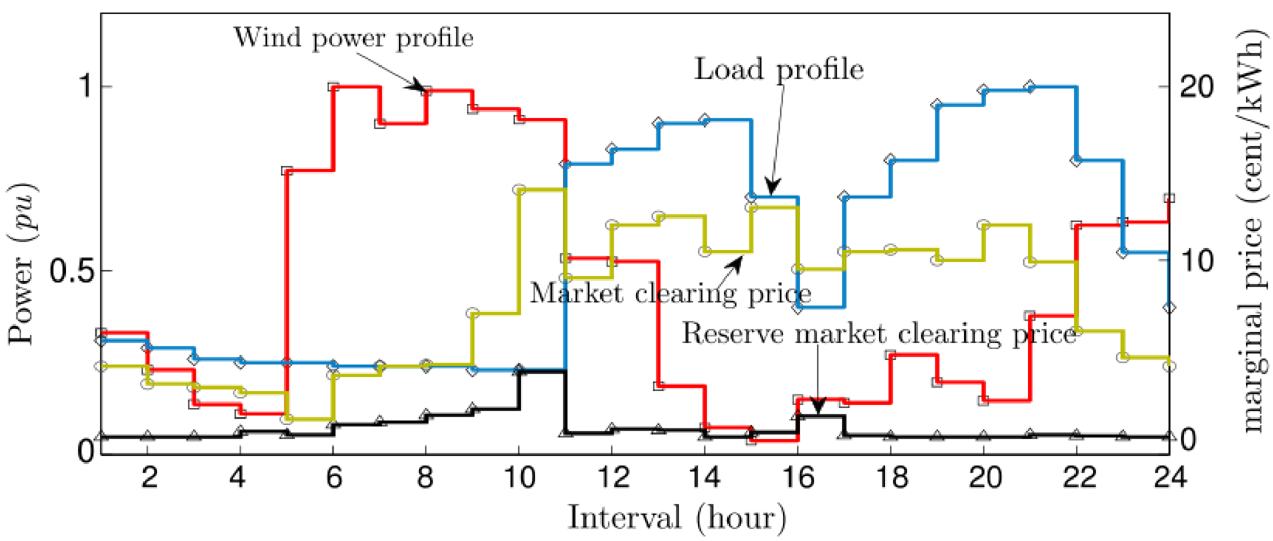

Fig. 3 *Forecasted Wind, Load, MCP and reserve MCP*

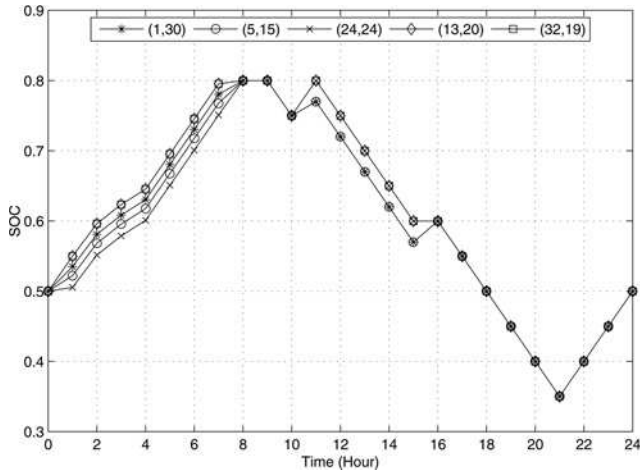

Fig. 4 *SOC profile of WPP located at different buses*

location of WPP, if ancillary services are not taken into account. Because, optimal solution in this case is not limited by network constraints. Therefore, hypothesis (i) is justified.

When ancillary benefit is considered in the objective function along with other objectives; objective function will depend upon location of WPP in the network. Because, voltage improvement and loss reduction in the network is dependent on location of WPP in the network. Hence, schedule will be modified with varying location of WPP and hence hypothesis (ii) is proved. Various SOC profile of WPP-BSD system is shown in Fig. 4. Since, wind generation and MCP day ahead forecast are negatively co-related for the scheduled day; BSD will charge during the initial hours, when MCP is low and wind generation is high. As seen from the figure, charging and discharging pattern of BSD is independent of location of WPP; but, SOC and hence the schedule gets modified for various location of WPP for maximising profit obtained by selling ancillary benefit.

Improvement in network loss for various placement of WPP is shown in Table 1. 'Cost of loss reduction' is reduction in cost of power purchase by DSO. Total loss reduction for collocated placement is comparatively small than its non-collocated counterpart. Because of non-colocation of WPP, although the schedule remains unchanged with WPP located at (24, 24) and (32, 19); loss reduction and its associated cost is high in the latter case. When WPP is located at (1, 30); WTG can't participate in improving performance on the network. Average improvement in voltage in this method considering random location of WPPs and average improvement in voltage profile is shown in Fig. 5. Node voltage represents, average of the range of voltage considered in pu. Average improvement for non-colocation can be seen in this figure.

Table 1 Loss reduction with WTG and BES placed at randomly chosen buses

(bus_w, bus_b) →	(1, 30)	(5, 15)	(24, 24)	(13, 20)	(32, 19)
total loss reduction, kWh	495.45	891.30	363.20	548.76	1025.20
cost of loss reduction, $	58.32	98.92	36.46	61.57	104.64

As it has been discussed earlier, proposed methodology can be used as an 'offline tool' for determining relative optimal location of WTG and BSD in the network. In the similar line, location of WTG and BSD is varied [according to assumption (iv)] throughout all the buses of the network to calculate the profit in each case. Thus a profit map for the day on which schedule is performed has been generated (see Fig. 6). Optimal location of WTG and BSD in the network cannot be determined considering a day's schedule. The objective of this paper is to find out relative location of BSD and WTG in the network for maximising the profit under the proposed scenario. Net profit generated from the network is low; when either of the WTG or BSD is located at node 19 or 23. Because, this node is physically located close to the substation. Hence, the profit map is dependent upon the network topology and load profile. If WTG and BSD are placed relatively far away in radial distribution network with respect to both network topology and loading, proposed strategy will generate maximum benefit; which can be called as 'non-adjacency'.

If one of the WTG or BSD is located near to substation, improvement in network performance gets reduced; since, the element placed near to substation will not contribute to network performance improvement. This condition can be termed as, 'substation effect'. Hence, WTG and BSD are needed to be placed relatively away from the substation and also from each other. Although net profit increases, when, collocated WTG and BSD are placed away from the substation (with respect to network topology and network loading); but, it can be observed that profit will be higher for corresponding non-collocated case.

3.2 Profit sharing among multiple WPP

Profit earned in independent (method B1) and cooperative (method B2) scheduling are compared in this section. Net profit generated depends upon location of WTG and BSD of individual WPPs in the network. When all the players schedule independently to maximise individual profit, profit by selling ancillary services does not get maximised, as the schedules of other players are not considered in this case. When the players schedule cooperatively, to improve ancillary benefit of all the players, revenue generated

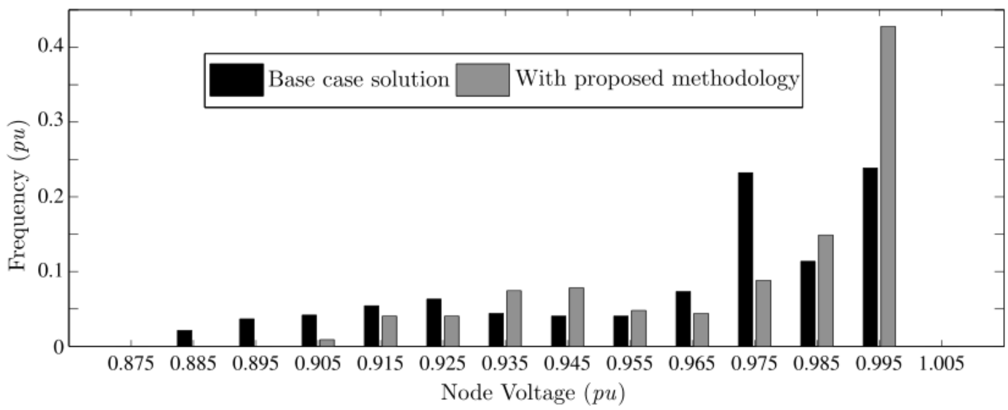

Fig. 5 *Improvement in voltage with proposed methodology; Y-axis represents frequency of occurrence of a certain range of voltage in pu*

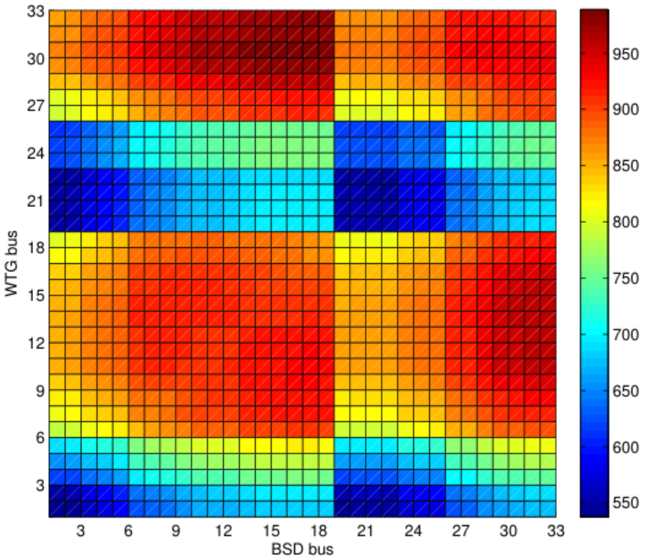

Fig. 6 *Profit map for the distribution network obtained for 24-hour scheduling; benefit of non-collocated placement in terms of overall profit, can be seen from this map*

by selling the power into the energy market reduces. Simulation result shows, comparative improvement in total network profit when method B1 has been chosen for scheduling compared with method B2. Two different pair of locations are arbitrarily chosen for WPPs. They are Pair 1: {(5, 12), (29, 19)}, and Pair 2: {(15, 22), (12, 30)}. Elements of pair 1 and pair 2 are location of individual WPPs (WPP1 and WPP2).

As seen from Fig. 7, combined net profit of all players in method B2 gets improved compared with the same in method B1. However in method B2, individual BSDs are no longer independently scheduled; hence, individual profit is no-longer maximised. Profit sharing among multiple WPPs, and total revenue and ancillary benefit generated are dependent on relative location of WTG and BSD of individual WPP. Hence, profit making ability of individual players are also depend upon relative location of WTG and BSD. Hence, as discussion in previous section, relative non-colocation of WTGs and BSDs are required for maximising ancillary benefit. All WPPs may decide location of individual WPP to reach a common goal of ancillary benefit maximisation. This can be independent of whether method B1 or B2 is sought.

3.3 Joint-scheduling of energy and reserve

The value of μ and σ of error distribution function are taken as 0 and 103.784 kW $(= 800\ \text{kW} \times 12.973\%)$ respectively. In the given problem, assumptions made for calculation of reserve requirement

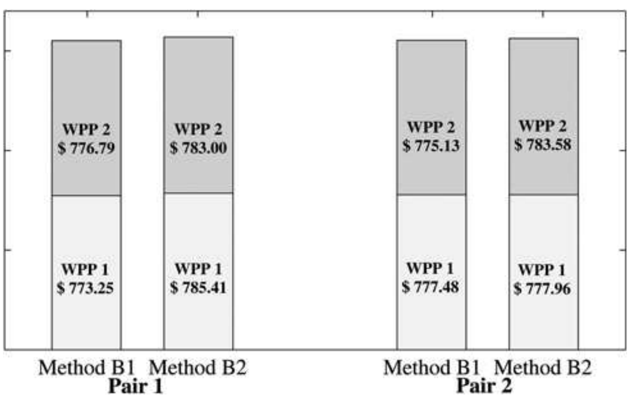

Fig. 7 *Profit sharing among multiple WPPs when WPPs schedule independently (method B1) or cooperatively (method B2)*

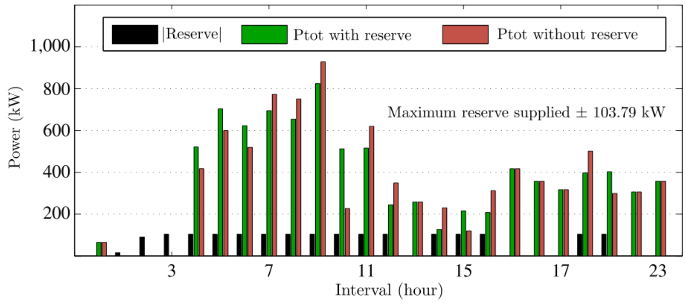

Fig. 8 *Hourly DA schedule with and without reserve*

from the battery results in transformation of probabilistic model into deterministic model. Moreover, actual reserve margin available for risk mitigation is always lesser than or equal to the schedule; because, reserve power has been considered as independent and identically distributed (i.i.d.) random variable.

Fig. 8 depicts a comparison of energy schedule of WPP with and without reserve. Reserve provided can be positive or negative, implying that BSDs can charge and discharge as a part of reserve schedule. Total power supplied during low marginal price hours are higher in this case. Therefore, the operational profit has been reduced. Reserve power requirement depends upon forecasted reserve MCP and MCP. Reserve power supplied by BSD is considered as an i.i.d., and so, actual reserve available depends upon SOC at the end of previous time instant. The proposed reserve allocation scheme is also applicable for multiple player scenario.

In the current scenario, reserve power demand at most of the time is limited by standard deviation of error in forecast uncertainty. By increasing the limit of reserve power supplied, WPP will becomes less vulnerable to the forecast error. However, it will negatively affect profit making ability of WPP in energy market. Higher reserve requirement have lower probability of occurrence; hence, choosing σ-limit for reserve helps WPP by not impeding its profit making ability.

4 Conclusion

DSO wants to improve network performance and reduce cost of power production; and WPP can improve the network performance by non-collocating self owned WTG and BSDs. This leads to selling of these two performance improvement metrics by WPP to the DSO as ancillary benefit. In this regard, an assessment of scheduling of non-collocated WPP is done in this paper. The proposed formulation have three objectives: (i) maximising the profit by selling wind energy into the market, (ii) maximising social benefit by providing loss reduction and voltage improvement, and (iii) minimising risk of wind power forecast uncertainty on schedule by providing reserve. Objective function along with other operational constraint constitutes the non-linear programming (NLP) problem.

With incorporation of network benefit as ancillary services into the objective function, the schedule will be modified; leading to performance improvement of the network. Location of WTG and BSD of a WPP are varied to generate a 'Profit map'; which shows the importance of relative location of WTGs and BSDs in the network for maximisation of ancillary benefit. It has been observed that while WTG and BSD are located away from each other and from the substation, the proposed scheme generates higher profit. Multiple WPPs can independently/cooperatively schedule themselves for optimal network performance improvement and individual and/or global profit. Sharing of ancillary benefit in terms of loss reduction and network voltage improvement among multiple WPPs are reported. It can be seen that, cooperative scheduling generates higher social benefit of all WPPs; but individual profit is no longer maximised. The scheduling strategy is applicable, even if a WPP owns multiple WTG and BSDs.

Assuming error in the forecast has normal distribution, and reserve is required for reduction of risk because of forecast error; objective function is modified to provide a Joint-Schedule. It has been observed that supplying reserve from self owned units reduces net profit by selling power into DA energy market. However in this process, WPP can maximise its net profit, while minimising its risk.

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7 Appendix

7.1 Appendix 1

To incorporate charging and discharging efficiency of the battery and converter efficiency; integer variables are required to be introduced in the optimisation problem. Hence, the mathematical model (1)–(17) will be modified as follows

$$u(i, t) \in \{-1, 0, 1\} \quad \forall i, t$$

where, $u(i, t)$ is an integer variable. If, $u(i, t) = 1$, battery is charging; $= 0$, battery is idle; and $= -1$, battery is discharging. Constraint (1) is modified to,

$$\mathrm{Re}_t = \mathrm{mcp}(t) \sum_{\forall i} \{P^w(i, t) - P^b(i, t) \cdot u(i, t)\} \quad \forall t$$

Constraint (9) will be modified to,

$$\mathrm{SOC}(i, t) = \mathrm{SOC}(i, t-1) + \frac{P^b(i, t)}{2 \cdot \mathrm{Cap}_b^i}\left[\delta \cdot \eta_{\mathrm{inj}} \cdot \{u(i, t) + 1\} + \frac{\{u(i, t) - 1\}}{\delta \cdot \eta_{\mathrm{ext}}}\right] \quad \forall t$$

where, δ is the converter efficiency, η_{inj} and η_{ext} are charging and dis-charging efficiency of the battery. Constraint (12) will be modified to,

$$0 \le P^b(i, t) \le \mathrm{Rat}_i^b \quad \forall t$$

Similarly, the mathematical model for calculating the joint-schedule of energy and reserve can be modified, considering battery and converter efficiency to be less than 100%.

7.2 Appendix 2

Error in hourly DA forecast of wind power follows Normal distribution, $\mathcal{N}(\mu, \sigma^2)$, with constant μ (=0) and constant σ. In a probabilistic framework, net profit by selling the power in the energy and reserve market are required to be 'joint optimised'.

In the absence of reserve, to meet the schedule for profit maximisation in the energy market, WPP can buy reserve from the DA reserve market at synchronised day ahead reserve MCP (*rmcp* (*t*)). For simplicity, it has been assumed that reserve power supplied is i.i.d. random variable and with normal distribution. Since, summation of sequence of i.i.d. never converges; hence, to completely eliminate the forecast uncertainty, reserve requirement is infinite. In this case, reserve power ($x(t) > 0$) has to be supplied so that conditional probability $\prod_t$ P($x(t)$) of not meeting the schedule (see Fig. 9) is minimised. The probability that the reserve being insufficient is shown in (30).

$$P(x(t)) = 2 \times \int_{x(t)}^{\infty} A \cdot \exp(-B \cdot (x(t))^2) \tag{30}$$

The equation depicting total probability of reserve being not sufficient to meet the DA schedule for all intervals is shown below. Since, reserve requirement is independent for all the hours, probability of independent events are multiplied to obtain the total probability.

$$\prod_t P(x(t)) = \prod_t 2 \times \left[\int_{x(t)}^{\infty} A \cdot \exp(-B \cdot (x(t))^2) \right] \tag{31}$$

Equation (31) can be rewritten as,

$$\prod_t P(x(t)) = \int \int \cdots \left(K_1 \exp\left(\sum_t -K_2 x(t)^2 \right) \right) \tag{32}$$

where, K_1, K_2 (≥ 0) are constants. To minimise the total probability, $\sum_t x(t)^2$ is required to be maximised. Moreover, the reserve power to be supplied has to be higher during high DA marginal price hours. Overall profit function is rewritten as follows

$$RP(X) = \sum_t \underbrace{\gamma(t) \cdot x(t)^2 \cdot \mathrm{mcp}(t)}_{\text{Benefit}} - \underbrace{x(t) \cdot \mathrm{rmcp}(t)}_{\text{Cost}} \tag{33}$$

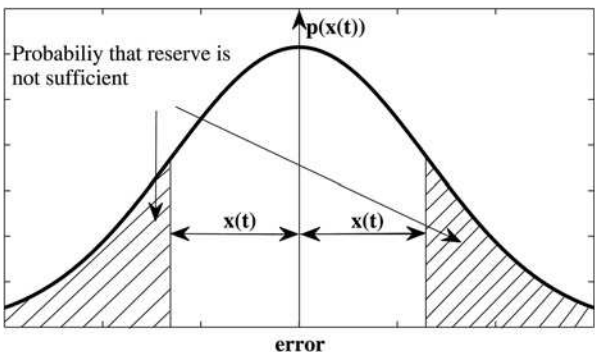

Fig. 9 *Error in wind power forecast and reserve*

This way the probabilistic model has been transformed into deterministic model. In (33), $\gamma(t)$ (≥ 0) is defined as the cost-equivalent conversion coefficient and $X = \{x(t)\}$ is reserve power vector. The profit obtained by selling 1 kW of reserve power is considered to be the break-even point (BEP), (i.e. $RP(X) = 0$).

$$\gamma(t) = \frac{\mathrm{rmcp}(t)}{x(t)|_{\mathrm{BEP}} \cdot \mathrm{mcp}(t)}$$

The necessary condition for optimal solution of profit function form providing reserve is $\left.\frac{\partial RP}{\partial x(t)}\right|_{x(t)^*} = 0$. Which implies,

$$\frac{\partial RP}{\partial x(t)}|_{x(t)^*} = 2\gamma(t)\mathrm{mcp}(t) \cdot x(t)^* - \mathrm{rmcp}(t) = 0 \quad \forall t$$

$$\Rightarrow x(t)^* = \frac{\mathrm{rmcp}(t)}{2 \cdot \gamma(t) \cdot \mathrm{mcp}(t)}$$

$$\text{And,} \quad \left.\frac{\partial^2 RP}{\partial x(t)^2}\right|_{x(t)^*} = 2\gamma(t) \cdot \mathrm{mcp}(t) \quad \geq 0 \quad \forall t$$

This shows that, given objective function has only one minima; and maximum values depend on operational limits. Proposed objective function for reserve schedule has been incorporated with (6) for Joint-Scheduling.

7.3 Appendix 3

Line and Peak Load data for 33-Node radial distribution network

Branch number	Sending end node	Receiving end node	*R*, Ω	*X*, Ω	Receiving end node	
					PLp, kW	QLp, kVAr
1	1	2	0.0922	0.0470	100	60
2	2	3	0.4930	0.2511	90	40
3	3	4	0.3660	0.1864	120	80
4	4	5	0.3811	0.1941	60	30
5	5	6	0.8190	0.7070	60	20
6	6	7	0.1872	0.6188	200	100
7	7	8	0.7114	0.2351	200	100
8	8	9	1.0300	0.7400	60	20
9	9	10	1.0440	0.7400	60	20
10	10	11	0.1966	0.0650	45	30
11	11	12	0.3744	0.1238	60	35
12	12	13	1.4680	1.1550	60	35
13	13	14	0.5416	0.7129	120	80
14	14	15	0.5910	0.5260	60	10
15	15	16	0.7463	0.5450	60	20
16	16	17	1.2890	1.7210	60	20
17	17	18	0.7320	0.5740	90	40
18	2	19	0.1640	0.1565	90	40
19	19	20	1.5042	1.3554	90	40
20	20	21	0.4095	0.4784	90	40
21	21	22	0.7089	0.9373	90	40
22	3	23	0.4512	0.3083	90	50
23	23	24	0.8980	0.7091	420	200
24	24	25	0.8960	0.7011	420	200
25	6	26	0.2030	0.1034	60	25
26	26	27	0.2842	0.1447	60	25
27	27	28	1.0590	0.9337	60	20
28	28	29	0.8042	0.7006	120	70
29	29	30	0.5075	0.2585	200	600
30	30	31	0.9744	0.9630	150	70
31	31	32	0.3105	0.3619	210	100
32	32	33	0.3410	0.5302	60	40