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Premium power investment strategy utilizing the economy of scale of custom power devices

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ABSTRACT

The possible existence of 'economy of scale' in manufacturing poor power quality (PQ) mitigation devices motivates customers to participate in a common mitigation solution. A custom power park (CPP) is an option where the CPP operator offers a set of custom solutions to a group of customers. Given that these custom solutions comprise one or many custom power devices, both CPP operators and the customers are expected to coordinate to obtain individual mitigation device ratings while maximizing the overall utility of customers. Here, CPP operators would calculate ratings of custom solutions and associated unit cost for the minimal total cost, while the customers are expected to select custom solutions to maximize their overall benefit. Furthermore, the customers utilize their willingness-to-pay function in this process and strictly participate in this arrangement if the utility received is more than their self-generated non-negative utility. Without the CPP operator, customers would form a CPP-like arrangement to recover the cost of investment and operation and maintenance. This combinatorial problem has been solved in two stages, involving the calculation of independently generated utility in the first stage and the overall CPP designing problem in the second stage. While customer and operator-side nonlinearities in the cost functions have been suitably discretized, the proposed methodology ensures that the solution space remains intact. The proposed method is illustrated using three devices, three custom solutions, and three customers.

1. Introduction

With ever-growing sensitive and critical equipments connected, power quality (PQ) issues are gaining immense significance [1–3]. Several poor PQ mitigation devices have been developed in recent years utilizing the concept of custom power devices (FACTS devices in transmission network) [4]. Here, two or more of such devices can operate in unison to ensure that even during faults and power interruptions, customers' voltage profiles stay within operational limits [5]. These power electronic devices can either employ network reconfiguration or voltage/current compensation techniques in this endeavor [6].

Because the requirement of a better PQ is related to customers' perceived utility, a system-wide investment in premium power and uniform distribution of costs might not lead to a socially justifiable outcome [7]. Improved PQ is essential for industrial customers who do not have the provision to augment the behavior of their sensitive and critical equipment [8]. One of the possible solutions for PQ improvement could be based on utilizing a game theory-based approach (discussed in [7,9] for voltage sag performance improvement), where the customers themselves generate mitigation solution provision. As indicated in [9], mitigation resources can be provided, managed, and priced by external organizations, which can fall under the custom power park (CPP) arrangement. The use of CPP for PQ improvement is widely discussed in the literature (refer to [5] for a summary of the draft IEEE P1409).

As shown in Fig. 1, Distribution-STATCOMs (d-STATCOM) protect the sensitive loads by ensuring availability of distortion-free voltages [10]. Distribution dynamic voltage restorers (DVRs) can be used to compensate voltage only to a set of sensitive loads, ensuring excludability of mitigation solution provisions. The make-before-break scheme ensures a smooth transition of critical loads into the alternative feeder using a static transfer switch (STS) during supply disturbances [11]. Non-custom power devices such as a backup diesel generator (DGs) or battery storage devices (BSDs) may also be used to ensure continuity of supply during outages of both the feeders. In this arrangement, all the loads connected enjoy the benefit of improved power quality, and

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Nomenclature

Sets

m	Set of discretized average cost components of mitigation devices ($\in \{1, 2, \ldots, M\}$).
n	Set of mitigation devices ($\in \{1, 2, \ldots, N\}$).
o	Set of custom solutions ($\in \{1, 2, \ldots, O\}$).
p	Set of various customers ($\in \{1, 2, \ldots, P\}$).
q	Set representing components of the piece-wise constant benefit function ($\in \{1, 2, \ldots, Q\}$).

Parameters

$\beta_{n,o}$	Binary number signifying whether a mitigation device, n, will be a part of the custom solution, o.
ϵ	A positive real number close to zero.
$\xi_{n,o}$	Binary number signifying types of a mitigation device, n, which will be a part of the custom solution, o.
$d^{S}_{m,n}$	Rating of the device, n, if the component, m, is chosen.
$l^{lo}_{p,q}$	Lower limit of the quality grade received by the customer, p, for the piece-wise component, q, of the benefit function.
$l^{up}_{p,q}$	Upper limit of the quality grade received by the customer, p, for the piece-wise component, q, of the benefit function.
$\mathcal{M}$	An arbitrarily large positive real number.
Qlty^{D}_{p}	Quality grade demanded by the customer, p.
Qlty^{S}_{n}	Quality grade provided by the mitigation device, n.
Qty^{D}_{p}	Device rating demanded by the customer, p.
$\mathrm{Ret}^{D}_{p,q}$	The average benefit received by the customer, p, if the piece-wise component q of the benefit function is chosen.
$\mathrm{Uni}^{S}_{m,n}$	Average unit cost of device, n, if the component, m, is chosen.

Variables

$\alpha_{m,n}$	Binary variables identifying whether the average cost component, m, of the mitigation device, n, will be selected.
$\gamma_{o,p}$	Binary variables identifying whether a custom solution, o, will be chosen by the customer, p.
$\delta_{p,q}$	Binary variables identifying whether the quality grade supplied to a customer, p, satisfies relevant lower limit of the associated piece-wise constant component, q, of its benefit function.
$\theta_{n,o,p,q}$	The result of multiplication of certain binary variables.
$\mu_{m,n,o,p}$	The result of multiplication of certain binary variables.
$\phi_{p,q}$	Binary variable identifying whether the quality grade supplied to a customer, p, satisfies the upper limit of the associated piece-wise constant component, q, of the benefit function.
$\psi_{n,o,p}$	The result of multiplication of certain binary variables.
Cst^{S}_{n}	Aggregated cost associated with the mitigation device, n.
Qlty^{S}_{o}	Aggregated quality grade received from the custom solution o.
QtyA^{S}_{o}	Alternative device rating available from the custom solution, o.
Qty^{S}_{n}	The device rating associated with the mitigation device, n.
Qty^{S}_{o}	The device rating available from the custom solution, o.
$\mathrm{Qty}^{S}_{n,o}$	The device rating of the mitigation device, n, that will be a part of the custom solution, o.
Tot^{D}_{p}	Aggregated benefit received by the customer, p.
Unilat_{p}	Utility that can be unilaterally generated by the customer, p.

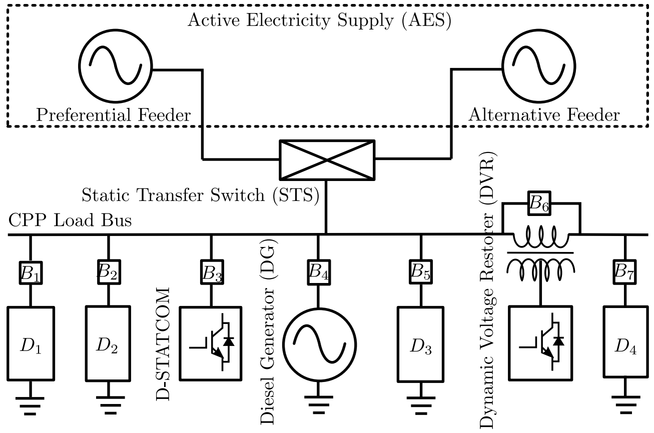

Fig. 1. A single line diagram of typical CPP.

the PQ of each of the loads is individually customizable and is usually termed as CPP [12].

Regarding the CPP, effective detection and transfer logic are required to facilitate custom PQ for an individual customer. All customers in a CPP (D_1, D_2, D_3 and D_4) receive the PQ superior to what they receive from the utility (through preferential or alternative feeder), as shown in Fig. 1. Here, all the customers and custom power devices are connected through breakers, B_1–B_7. The rating of the relevant custom power devices should be a linear sum of the device ratings of customers being served. An extended CPP concept to ensure better coordination among the custom power devices and loads through the PQ control center has been introduced in [13]. The design of a CPP to ensure the coordinated operation of custom power devices is discussed in [14]. A competing research concept, known as ‘power quality park’ [15], also exists in the literature.

A typical PQ improvement mitigation device installation plan begins with a cost–benefit analysis. Evaluation of financial incentives received by the customers with various mitigation devices is presented in [16]. Allocation of FACTS-based devices among the customers has been considered in [17,18]. A ‘nested logic’ based approach for optimal selection of mitigation devices has been considered in [19]. Optimal mitigation solution provision for PQ performance index improvement has been considered in [20]. Individual process failure characteristics within a typical industrial plant have been considered for the cost–benefit analysis [21]. Given that a mitigation device serves a multitude of objectives, the use of multiple objectives in an optimization framework has been considered in [22]. Placement of mitigation solution within the distribution network to improve PQ performance has been considered in [23]. A premium power investment scheme and an optimal investment strategy based on disappointment-rejoicing theory are studied in [24].

Now, in the case of a dis-economy of scale, where the cost of individual custom power devices monotonically increases with their ratings, the customers will be strongly incentivized to install mitigation devices by themselves [9]. However, the cost function available in the literature for typical FACTS devices indicates the cost characteristics to be otherwise ([17] can be referred to for the cost function for a typical custom power device). Additionally, the economy of scale also exists in the production cost function of DGs (otherwise, the demanding customers are better off installing DGs within their premises).

Given the customer's willingness-to-pay or benefit function and the cost function of the custom power devices, one needs to find out the solution strategy that maximizes the overall utility (= benefit - cost) generated. In this regard, the CPP operator is expected to identify the optimal sizing of custom power devices comprising the custom solutions. The customers are expected to identify the custom solution that maximizes their benefit. Furthermore, the customers should strictly receive more utility than what they can generate on their own. However, unlike [9], the cost of mitigation solution provision will be distributed among the customers based on their device ratings since the free-riding utility, in this case, will be zero. Because of the absence of the free-riding utility, customers themselves can provide the CPP-like arrangement, or, under the regulator's recommendation, the utility companies, or, a third party, can carry out a similar exercise, where the customers will be liable to pay their share of installation and operation cost. Unlike in a traditional CPP, where the customers are expected to receive better PQ at the expense of a premium, the customers become the investors in this CPP-like arrangement. Here, it has been assumed that the customers are free to choose among the available custom solutions without discrimination. Therefore, the scope of the CPP-like arrangement is limited to mitigating voltage sags/swells and momentary/permanent interruptions which could be suitably expanded for other PQ-related problems.

The contribution of this paper is two-fold:

(i) An optimal design and cost-sharing methodology of mitigation solutions within a CPP-like arrangement is the primary objective of this paper. The methodology ensures that the customers receive more utility than what they can generate on their own. Both cost and benefit functions are suitably discretized, and consequently, the overall non-linear problem has been converted to become a mixed-integer linear programming problem. The computation complexity of the overall optimization problem has been discussed. While the proposed problem is a planning problem, it has been shown that the overall methodology can be parallelized, significantly reducing the computation time.

(ii) The proposed methodology has been demonstrated using three devices, three custom solutions, and three customer problem. It has been shown that the proposed discretization has no impact on the quality of the solution received, and the methodology can be used for a non-linear mixed integer optimization problem with similar properties. Methods to incorporate devices, the ratings of which are not customizable, have also been described.

The rest of the paper is organized as follows. Operating principles of custom power devices constituting various custom solutions are discussed in Section 2. The development of the utility-maximizing CPP design and the mechanism for an appropriate distribution of investment costs among the customers are discussed in Section 3. The proposed methodology has been demonstrated using a numerical example and is presented in Section 4. Section 5 summarizes this paper.

2. The relationship among various custom solutions, custom power devices and customer demands

Three different custom solutions are possible based on the established coordination strategies among the custom power devices (as discussed in [6]). A brief theoretical description of these custom solutions is given below:

2.1. Custom solution A

The use of both STS and d-STATCOM ensures the availability of a harmonic-free, balanced power supply to the loads. This basic solution is available to all the customers participating in the CPP. The STS in this quality grade is able to detect sag/swell events and rapidly transfer the loads into the healthy alternative feeder within 4–10 ms under the make-before-break scheme. This way, the duration of the experienced sag/swell event will be significantly reduced.

2.2. Custom solution AA

In the event of sag/swell/failure in both the feeders, the backup DG can be brought in immediately isolating the loads in Custom Solution AA and higher. Therefore, this custom solution receives all the performance improvement benefits of custom solution A; additionally, it is long interruption-free.

2.3. Custom solution AAA

Both the custom solutions A and AA suffer from voltage sag/swell events during the transfer of loads from the faulted feeder through STS. The incorporation of DVR in this custom solution turns the loads to become sag/swell free.

In summary, during normal operating conditions, the backup DG stays off, and custom solutions A, AA, and AAA enjoy similar services. While custom solutions A and AA will experience voltage sags for a very short duration, loads enjoying custom solution AAA are protected from sags utilizing DVR. In the event both preferential and alternative feeders are lost, loads enjoying the custom solution AA will be continued to be served through backup diesel generators. A state-flow chart representing sequential isolation and re-connection of breakers is available in [25].

However, given the expensiveness of these custom power solutions and load demands constitute both sensitive and non-sensitive parts (where the impacts of poor PQ can significantly differ), customers need to make a judicious decision in the optimal custom solution investment. There are two ways in which customers can decide on an improved PQ solution:

(i) The customers can separate out their sensitive and non-sensitive components and select custom solutions separately for both components. However, such segregation may not always be practical due to regulatory challenges.

(ii) Customers are not able to segregate their overall loads, and both sensitive and non-sensitive components enjoy a similar benefit. Since both the load components enjoy improved PQ, while the protection of the sensitive components is only beneficial, given a limited budget, overall custom solution provision to the sensitive component will be significantly reduced.

3. Problem statement and model description

Here, custom power devices are identified as 'devices' to ensure the genericness of the problem and associated solution methodology.

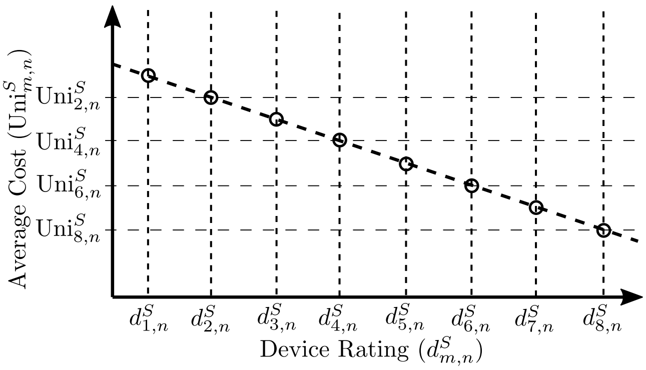

Fig. 2. Typical model representing economy of scale in the production process of the devices.

3.1. Cost and benefit function models

A typical model of linear and monotonically decreasing average manufacturing cost, representing the economy of scale [17] is shown in Fig. 2. Cost function can assume various other characteristics and has been discretized here. The resulting problem is of mixed-integer type, which avoids possible non-linearities in the cost function.

Suppose, the cost function of the device 'n', as shown in Fig. 2, is discretized into 'm' distinct elements. If the said device is selected, it is imminent that for each of the devices, at most, one of these segments will remain active (see, (1b)). Consequently, one can obtain the following set of constraints:

$$\mathrm{Qty}^S_n = \sum_{m=1}^{M} d^S_{m,n}\alpha_{m,n}; \quad \forall n \tag{1a}$$

$$\sum_{m=1}^{M} \alpha_{m,n} \leq 1; \quad \forall n; \quad \alpha_{m,n} \in \{0,1\}; \quad \forall m,n \tag{1b}$$

$$\mathrm{Cst}^S_n = \sum_{m=1}^{M} \mathrm{Uni}^S_{m,n} d^S_{m,n}\alpha_{m,n}; \quad \forall n \tag{1c}$$

Here, $\alpha_{m,n}$ are binary variables identifying selection status of discrete element m. Hence, Qty^S_n identifies the rating of device n (see, (1a)), and the total cost of the device is captured using Cst^S_n (see, (1c)).

As described in the previous section, each of the custom solutions comprises of various combination of these devices; and only a certain set of custom solutions can be feasible. These feasible custom solutions are identified by β. Here, $\beta_{n,o} = 1$, if the device 'n' constitutes the custom solution 'o', otherwise, $\beta_{n,o} = 0$. Notably, $\beta_{n,o}$ is provided by the planner to identify custom devices with custom solutions. Given, one device can serve multiple custom solutions, the overall device rating will be sum of the rating of individual custom solutions, and is shown in (2a). As shown in (2b), The quality grade of the custom solution 'o', (Qlty^S_o) is assumed to be a linear sum[1] of the quality grades of constituting devices, Qlty^S_n. It is imminent that the ratings of all the devices contributing to a custom solution need to be equal, which will also be equal to the device rating of the custom solution, 'o,' as shown (2c). However, with devices, such as, d-STATCOM, ratings of custom power devices may not remain comparable. For example, if d-STATCOM is expected to inject only reactive power into the grid, alternative ratings of the custom power devices will be used. Here, $\xi_{n,o} = 1$, if mitigation devices are series connected, else, device rating would match alternative quantity available at the custom solution, QtyA^S_o. Given, d-STATCOMs are expected to ensure distortion free voltage, it should provide both active and reactive power into the grid. Mathematically,

$$\mathrm{Qty}^S_n = \sum_{o=1}^{O} \mathrm{Qty}^S_{n,o}\beta_{n,o}; \quad \forall n \tag{2a}$$

$$\sum_{n=1}^{N} \mathrm{Qlty}^S_n \beta_{n,o} = \mathrm{Qlty}^S_o; \quad \forall o \tag{2b}$$

$$\left(\mathrm{Qty}^S_{n,o} - \mathrm{Qty}^S_o\right)\beta_{n,o}\xi_{n,o} + \left(\mathrm{Qty}^S_{n,o} - \mathrm{QtyA}^S_o\right)\beta_{n,o}(1-\xi_{n,o}) = 0;$$
$$\forall n,o; \; \beta_{n,o} \in \{0,1\}; \; \forall n,o; \; \xi_{n,o} \in \{0,1\}; \; \forall n,o \tag{2c}$$

Each customer (with or without suitably dividing itself into sensitive and non-sensitive components) 'p' requires an internal minimum quality grade of Qlty^D_p. As shown in (3a), for successful participation, the customer should receive a grade of more than its declared minimal value. The device rating of custom solutions needs to be equal to the linear sum of the device rating demanded by the customers being served. Here, $\gamma_{o,p}$ identifies if a custom solution 'o' is allocated to customer 'p', where, $\gamma_{o,p} = 1$ represents that custom solution 'o' serves associated customer 'p'. Furthermore, each of the customers can be served by at-most one custom solution (see, (3d)). Also, as shown in (3b), the rating of the custom solution should be equal to the ratings of the customer devices being protected. Also, as discussed earlier, customers' alternative device rating, QtyA^D_p, will be used to obtain an alternative device rating at the custom solution, QtyA^S_o (see, (3c)). Essentially,

$$\mathrm{Qlty}^S_p = \sum_{o=1}^{O} \mathrm{Qlty}^S_o \gamma_{o,p} \geq \mathrm{Qlty}^D_p \sum_{o=1}^{O} \gamma_{o,p}; \quad \forall p \tag{3a}$$

$$\mathrm{Qty}^S_o = \sum_{p=1}^{P} \mathrm{Qty}^D_p \gamma_{o,p}; \quad \forall o \tag{3b}$$

$$\mathrm{QtyA}^S_o = \sum_{p=1}^{P} \mathrm{QtyA}^D_p \gamma_{o,p}; \quad \forall o \tag{3c}$$

$$\sum_{o=1}^{O} \gamma_{o,p} \leq 1; \quad \forall p; \quad \gamma_{o,p} \in \{0,1\}; \quad \forall o,p \tag{3d}$$

Given the rating of a custom solution is equal to custom power devices ratings, and ratings of each of the custom solutions will be an algebraic sum of ratings of participating customers, the discretization points discussed in (1) need not be randomly generated. However, without solving the problem itself, it is challenging to determine participating customers to each custom solution, vis-á-vis the devices. If there are P numbers of participating customers, there will be exactly 2^P number of ways the customers will demand each of the custom solutions vis-á-vis the devices (after suitably accounting for alternative

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[1] Alternatively, quality grade of each of the custom solutions can be provided apriori, and indicated to the customers beforehand. It has also been assumed that benefits of multiple PQ solutions could be suitably aggregated, otherwise, multiple quality aspects could remain separated, and matched against customers preferences as needed while matching against individual budget constraints.

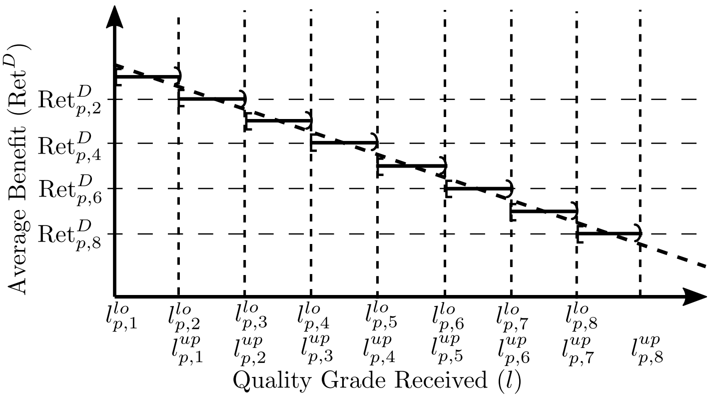

Fig. 3. Model representing decreasing average benefit received by customers.

rating requirements of individual devices), and, the ratings of the devices need to be discretized only at those 2^P $(= M)$ possible points.

While the quality grade received at the customers' premises is highly discretized (the quality grade received will be equal to one of the possible quality grades of custom solutions), treating the benefit function as a piece-wise constant (see Fig. 3) variable ensures computational simplicity. While a linearly decreasing average benefit curve with respect to the quality grade received is considered in Fig. 2 for representation (and has been derived to be a benefit function of voltage sag mitigation in [26]), non-linear benefit characteristics can be modeled in a similar way. Such a piece-wise constant segment representation, if chosen suitably, is expected to avoid the introduction of non-linearity into the problem.

Each of the piece-wise constant segments, 'q,' of the benefit function for the customer, 'p,' is bounded by $[l^{lo}_{p,q}, l^{up}_{p,q})$. Two variables, namely, $\delta_{p,q}$ and $\phi_{p,q}$, has been utilized to identify active segment of the benefit function for a particular customer. Here, if, $\delta_{p,q} = 1$, the received quality grade is higher than associated lower limit of the segment q (see, (4a)). Similarly, $\phi_{p,q} = 1$ when the received quality grade is lower than the upper limit of the segment q (see, (4b)). However, while designing segments of the piece-wise constant benefit function, one needs to ensure that the quality grade received does not reside at the limiting points of the segments to prevent erroneous outcomes. From Fig. 3, if $\mathrm{Ret}^D_{p,q}$ is the average benefit received by customer p with selection of segment q, total benefit received can be calculated using (4c). It is notable that these quality characteristics indirectly provide us with the budget of individual customers as a function of the quality grade received.[2] Therefore,

$$\delta_{p,q} = \begin{cases} 1 & \text{if } l^{lo}_{p,q} \le \mathrm{Qlty}^S_p \\ 0 & \text{if } l^{lo}_{p,q} > \mathrm{Qlty}^S_p \end{cases}; \quad \forall p, q \tag{4a}$$

$$\phi_{p,q} = \begin{cases} 1 & \text{if } \mathrm{Qlty}^S_p < l^{up}_{p,q} \\ 0 & \text{if } \mathrm{Qlty}^S_p \ge l^{up}_{p,q} \end{cases}; \quad \forall p, q \tag{4b}$$

$$\mathrm{Tot}^D_p = \sum_{q=1}^{Q} \mathrm{Ret}^D_{p,q} \mathrm{Qlty}^S_p \delta_{p,q} \phi_{p,q}; \quad \forall p \tag{4c}$$

$$\delta_{p,q} \in \{0, 1\}; \quad \phi_{p,q} \in \{0, 1\}; \quad \forall p, q \tag{4d}$$

To ensure successful participation of a customer in a custom solution, one needs to ensure that the utility received by a customer in a group is more than what it can generate on its own (Unilat_p), i.e., by installing mitigation devices within its own premises. It is also notable that $\mathrm{Unilat}_p \ge 0$. This constraint is mathematically expressed as follows:

$$\mathrm{Tot}^D_p - \sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{o=1}^{O} \mathrm{Uni}^S_{m,n} \mathrm{Qty}^D_p \alpha_{m,n} \beta_{n,o} \gamma_{o,p} \ge \mathrm{Unilat}_p \sum_{o=1}^{O} \gamma_{o,p} \ge 0; \forall p \tag{5}$$

The objective considered here is to maximize the aggregated utility generated by the group. Since $\mathrm{Unilat}_p \ge 0$, it can also be said that the utility generated is non-negative, ensuring the feasibility of the group formation.

$$\max \sum_{p=1}^{P} \mathrm{Tot}^D_p - \sum_{n=1}^{N} \mathrm{Cst}^S_n \tag{6}$$

3.2. The optimization problem

Aggregating all the constraints, one can transform the entire problem into (complexities introduced with alternative device ratings, various kinds of power quality related solutions are ignored for simplicity):

$$\max \sum_{n=1}^{N} \sum_{o=1}^{O} \sum_{p=1}^{P} \sum_{q=1}^{Q} \mathrm{Qlty}^S_n \mathrm{Ret}^D_{p,q} \theta_{n,o,p,q} - \sum_{m=1}^{M} \sum_{n=1}^{N} \mathrm{Uni}^S_{m,n} d^S_{m,n} \alpha_{m,n} \tag{7}$$

subject to,

$$\sum_{m=1}^{M} \alpha_{m,n} \le 1; \quad \forall n; \quad \alpha_{m,n} \in \{0, 1\}; \quad \forall m, n \tag{8}$$

$$\sum_{o=1}^{O} \gamma_{o,p} \le 1; \quad \forall p; \quad \gamma_{o,p} \in \{0, 1\}; \quad \forall o, p \tag{9}$$

$$\delta_{p,q} \in \{0, 1\}; \quad \phi_{p,q} \in \{0, 1\}; \quad \forall p, q \tag{10}$$

$$\sum_{m=1}^{M} d^S_{m,n} \alpha_{m,n} - \sum_{o=1}^{O} \mathrm{Qty}^S_{n,o} \beta_{n,o} \ge 0; \quad \forall n \tag{11}$$

$$\left(\mathrm{Qty}^S_{n,o} - \mathrm{Qty}^S_o \right) \beta_{n,o} \ge 0; \quad \forall n, o \tag{12}$$

$$\sum_{n=1}^{N} \sum_{o=1}^{O} \mathrm{Qlty}^S_n \psi_{n,o,p} \ge \sum_{o=1}^{O} \mathrm{Qlty}^D_p \gamma_{o,p}; \quad \forall p \tag{13}$$

$$\mathrm{Qty}^S_o = \sum_{p=1}^{P} \mathrm{Qty}^D_p \gamma_{o,p}; \quad \forall o \tag{14}$$

$$l^{lo}_{p,q} - \sum_{n=1}^{N} \sum_{o=1}^{O} \mathrm{Qlty}^S_n \psi_{n,o,p} \ge -\mathcal{M} \delta_{p,q} + \epsilon \left(1 - \delta_{p,q} \right); \quad \forall p, q \tag{15}$$

$$l^{lo}_{p,q} - \sum_{n=1}^{N} \sum_{o=1}^{O} \mathrm{Qlty}^S_n \psi_{n,o,p} \le \mathcal{M} \left(1 - \delta_{p,q} \right); \quad \forall p, q \tag{16}$$

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[2] The scope of power quality analysis can be wide due to the consideration of voltage sags/swells, harmonics, voltage unbalances, etc., and, one can weight each of the factors differently to generate a comprehensive power quality index so that impact of each factor may not interact with each other. Alternatively, the customers would provide the ratings of each of the components of power quality separately, which could also be included within the proposed framework without the loss the generality.

$$\sum_{n=1}^{N}\sum_{o=1}^{O} \mathrm{Qlty}_n^S \psi_{n,o,p} - l_{p,q}^{up} \geq -\mathcal{M}\phi_{p,q}; \quad \forall p, q \tag{17}$$

$$\sum_{n=1}^{N}\sum_{o=1}^{O} \mathrm{Qlty}_n^S \psi_{n,o,p} - l_{p,q}^{up} \leq \mathcal{M}\left(1 - \phi_{p,q}\right) - \epsilon\phi_{p,q}; \quad \forall p, q \tag{18}$$

$$\sum_{n=1}^{N}\sum_{o=1}^{O}\sum_{q=1}^{Q} \mathrm{Qlty}_n^S \mathrm{Ret}_{p,q}^D \theta_{n,o,p,q} - \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{o=1}^{O} \mathrm{Uni}_{m,n}^S \mathrm{Qty}_p^D \mu_{m,n,o,p} \geq \mathrm{Unilat}_p \sum_{o=1}^{O} \gamma_{o,p}; \quad \forall p \tag{19}$$

$$\mu_{m,n,o,p} = \alpha_{m,n}\beta_{n,o}\gamma_{o,p}; \quad \forall m, n, o, p \tag{20}$$

$$\theta_{n,o,p,q} = \beta_{n,o}\gamma_{o,p}\delta_{p,q}\phi_{p,q}; \quad \forall n, o, p, q \tag{21}$$

$$\psi_{n,o,p} = \beta_{n,o}\gamma_{o,p}; \quad \forall n, o, p \tag{22}$$

Nonlinearities present in the objective function (6), and the constraints (8)–(19), with multiplications of binary variables, could be suitably linearized. Linearization of multiplications of binary variables presented in Eqs. (20)–(22) can be explained using the following example:

$$\mu_{m,n,o,p} \geq 0; \quad \forall m, n, o, p \tag{23a}$$

$$\mu_{m,n,o,p} \leq \alpha_{m,n}; \quad \forall m, n, o, p \tag{23b}$$

$$\mu_{m,n,o,p} \leq \beta_{n,o}; \quad \forall m, n, o, p \tag{23c}$$

$$\mu_{m,n,o,p} \leq \gamma_{o,p}; \quad \forall m, n, o, p \tag{23d}$$

$$\mu_{m,n,o,p} \geq \alpha_{m,n} + \beta_{n,o} + \gamma_{o,p} - 2; \quad \forall m, n, o, p \tag{23e}$$

Thusly, (6)–(19) and linearized (20)–(22) together constitutes a mixed-integer linear problem.

Remark 1. If the quality grade available at the custom solution 'o' is less than the minimum quality grade demanded by the customer 'p,' then the associated $\gamma_{o,p} = 0$.

Remark 2. Suppose, $\max_{\forall m} d_{m,n}^S$ be the maximum device rating available from the device 'n'. Then, if, $\max_{\forall n} \max_{\forall m} d_{m,n}^S \beta_{n,o} \geq \mathrm{Qty}_p^D$, then $\gamma_{o,p} \geq 0$. Else, $\gamma_{o,p} = 0$.

Proofs of Remarks 1 and 2 are reasonably trivial and can be used to reduce the solution space significantly.

3.3. Solution strategy

Customers' self-generated utilities, Unilat_p ($\forall p$), are not known apriori and will be calculated first, as shown below:

$$\mathrm{Unilat}_p = \max \sum_{n=1}^{N}\sum_{o=1}^{O}\sum_{q=1}^{Q} \mathrm{Qlty}_n^S \mathrm{Ret}_{p,q}^D \theta_{n,o,p,q} - \sum_{m=1}^{M}\sum_{n=1}^{N} \mathrm{Uni}_{m,n}^S d_{m,n}^S \alpha_{m,n} \tag{24}$$

subject to, (8), (10)–(22), and

$$\sum_{o=1}^{O} \gamma_{o,p} \leq 1; \quad \gamma_{o,p} \in \{0, 1\}; \quad \forall o; \quad \gamma_{o,p'} = 0; \quad \forall p' \neq p \tag{25}$$

$$\sum_{n=1}^{N}\sum_{o=1}^{O}\sum_{q=1}^{Q} \mathrm{Qlty}_n^S \mathrm{Ret}_{p,q}^D \theta_{n,o,p,q} - \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{o=1}^{O} \mathrm{Uni}_{m,n}^S \mathrm{Qty}_p^D \mu_{m,n,o,p} \geq 0; \quad \forall p \tag{26}$$

Notably, 'P' number of problems solved in the first stage are independent of each other, and this way, the overall problem could be parallelized. Also, as discussed earlier, a number of discretization points for individual devices will also be limited, further limiting solution space. The optimization problem (7)–(22), embracing all the customers, is solved in the second stage of the two-stage problem.

Table 1
Unilaterally generated utilities.

Only customer 1 active	$\gamma_{3,1} = 1$	$\mathrm{Unilat}_1 = 3.07 \times 10^4$ MU
Only customer 2 active	$\gamma_{3,2} = 1$	$\mathrm{Unilat}_2 = 4.16 \times 10^4$ MU
Only customer 3 active	$\gamma_{3,3} = 1$	$\mathrm{Unilat}_3 = 8.02 \times 10^4$ MU

Table 2
Utility distribution.

Total		
16.05×10^4 MU		
Customer 1	Customer 2	Customer 3
3.08×10^4 MU	4.56×10^4 MU	8.40×10^4 MU

3.4. Complexity

Each of both the first and second stages of the optimization problem constitutes $2^P N + OP + 2QP$ number of binary variables (P=1 in the first stage). Because of linearization, an additional $|\beta|\, OP\left(1 + 2^P N + PQ\right)$ number of binary variables are introduced. This way, the second stage of the problem is exponential in complexity concerning the number of customers. However, the total number of the optimization problem to be solved is $P + 1$, and stage one of the problem is relatively easy to solve. Additionally, with the increasing number of customers, the complexity of the second-stage problem grows exponentially. Furthermore, an increasing number of piece-wise constant segments of the customer's benefit function polynomially increases the problem complexity.

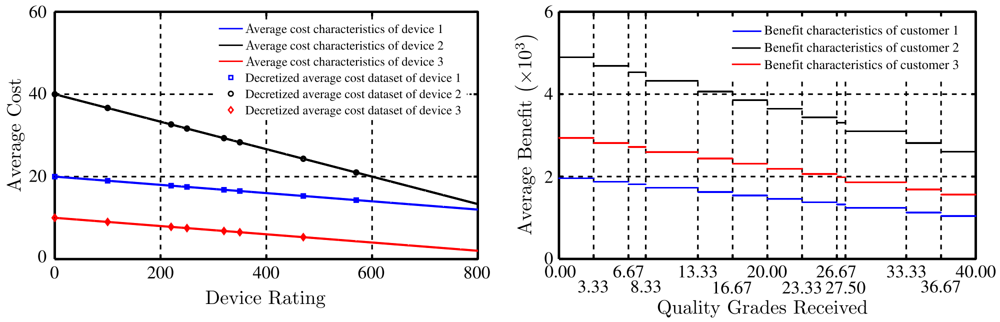
4. Illustration

The proposed methodology is illustrated by utilizing three devices, three custom solutions, and three customer problem, as shown in Fig. 4. The average cost and benefit functions are presented in Figs. 4a and 4b, respectively. In this problem, the values of $\mathcal{M}$ and ϵ are selected to be 1×10^4 and 1×10^{-6} respectively. Both the first and the second stages of the combinatorial problem are solved using SCIP (Solving Constraint Integer Programs) [27] mixed-integer programming solver from GAMS (General Algebraic Modeling System) [28]. The average solving time in both the stages for the given problem is 200 ms with an i5 processor and 8 GB RAM. Notably, the proposed CPP designing methodology can be equally applicable to various other engineering design problems.

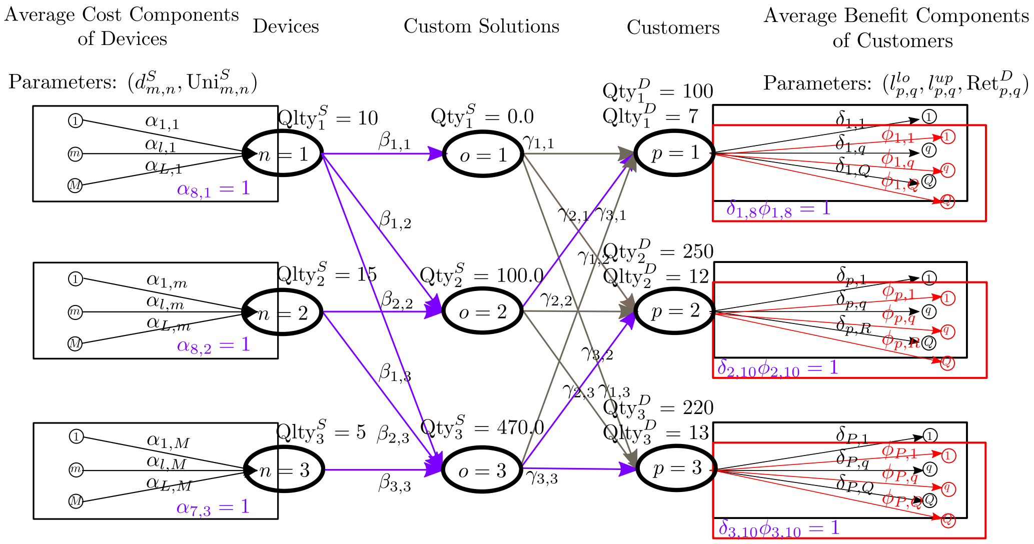
Outcomes of stage one optimization problems are presented in Table 1. Since the total cost of the custom solution with the highest quality grade is lower than the perceived benefit, customers tend to choose the highest quality grade solutions. However, the finite capacity constraint of the devices vis-á-vis the custom solutions, custom solutions received by individual customers may significantly differ subject to no reduction in utility received.

The utility distributed in stage two of the problem is given in Table 2, and the network representation of the problem is given in Fig. 4c. As it is desired, it can be seen that the process not only generates a higher utility than the cumulative sum of the utility generated by individual customers, each of the customers receives more utility than what they could generate individually. This way, the benefits received from the collaborative behavior to enjoy the benefits of 'economy of scale' is imminent here.

One important observation here is that the total utility received by a customer 'p' is given by $\sum_{n=1}^{N}\sum_{o=1}^{O}\sum_{q=1}^{Q} \mathrm{Qlty}_n^S \mathrm{Ret}_{p,q}^D \theta_{n,o,p,q} - \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{o=1}^{O} \mathrm{Uni}_{m,n}^S \mathrm{Qty}_p^D \alpha_{m,n}\beta_{n,o}\gamma_{o,p}$. Since, benefit functions of the customers are publicly known (privately known if the utility is carrying out this exercise), each of the customers are expected to invest $\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{o=1}^{O} \mathrm{Uni}_{m,n}^S \mathrm{Qty}_p^D \alpha_{m,n}\beta_{n,o}\gamma_{o,p}$ for the optimal quality grade received. Furthermore, $\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{o=1}^{O}\sum_{p=1}^{P} \mathrm{Uni}_{m,n}^S \mathrm{Qty}_p^D \alpha_{m,n}\beta_{n,o}\gamma_{o,p} = \sum_{m=1}^{M}\sum_{n=1}^{N} \mathrm{Uni}_{m,n}^S d_{m,n}^S \alpha_{m,n}$; that is, the investment cost is fully recovered from the customers. Also, it is expected that the requisite investment



(a) Devices cost characteristics. (b) Customer's benefit function.



(c) Optimal resource provision in a CPP.

Fig. 4. Results of the optimization problem.

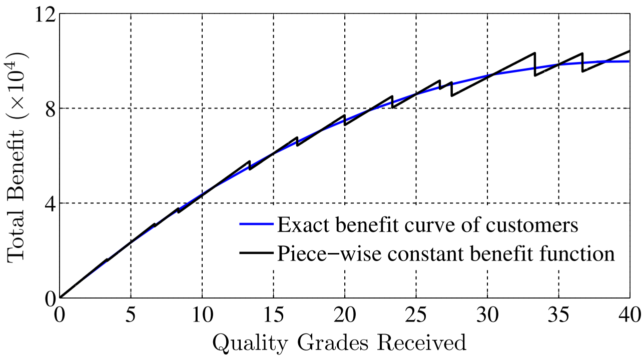


Fig. 5. Example total benefit received by customer 3.

in each of the devices is uniformly distributed among the benefiting customers.

The maximum device rating of a custom solution is given by $\min_n \beta_{n,o} \max_m d^S_{m,n}$. This way, the maximum device rating of the custom solution with the highest quality grade is limited to 470.0. Although all the customers would like to receive the highest possible quality grade, such a custom solution also has to satisfy the device ratings of all the customers simultaneously. This way, the device ratings of customers 2 and 3 can be served from the custom solution 3. Because of the relatively lower average benefit received, it is optimal to satisfy customer 1's demand from custom solution 2.

Owing to the utilization of piece-wise constant benefit functions, one can observe from Fig. 5 that the approximate benefit function can be significantly erroneous. However, although variables Qlty^S_p are treated as continuous in (4a), (4b) and (4c), the only feasible values it is allowed to take are $\left\{\sum_{n=1}^N \mathrm{Qlty}^S_n \beta_{n,o};\ \forall o\right\}$. These values are inherently discrete. Therefore, if one can ensure the total benefit received is equal to the approximated benefit at the quality grades $\left\{\sum_{n=1}^N \mathrm{Qlty}^S_n \beta_{n,o};\ \forall o\right\}$, the approximation will have no impact on the solution-space. Notably, the solution space will be further simplified with the replacement of the piece-wise benefit function with discretized quality grades available at the custom solution level. Also, piece-wise representation requires product of binary variables, $\delta_{p,q}$ and $\phi_{p,q}$, contrary to single binary variable requirement in (1c). The discussed piece-wise representation is merely to show the versatile way the proposed MINLP model could be converted to the MILP model.

Challenges will also arise if the relationship between the unit cost of devices and associated ratings becomes inherently discrete, and this way, (3b) may never be satisfied. A device with a discontinuous cost function can only be utilized if the rating of the device is strictly higher than the requisite rating of demanding customers. Additionally, to satisfy (3b), while keeping the total investment cost constant, one needs to modify the cost function in Fig. 4a in such a way that the rating of the custom power device coincides with the nearest of the 2^P ratings discussed earlier, with the new rating less than the actual rating of the devices. Furthermore, the unit average cost of the associated devices increases by the same factor to ensure the total cost of the devices is appropriately accounted for. This modification ensures that both the rating and cost of overrated devices are appropriately shared among the requisite customers.

5. Conclusion

The economy of scale in procuring the custom power devices may motivate the customers to participate in a common mitigation solution, and customers may seek a CPP-like approach. Possible customizability prevents the free-riding of individual customers in this arrangement. While the customers can set up this CPP-like approach themselves and conduct the CPP design process, the utility company may carry out this analysis on their behalf if they are unwilling to exchange their ratings and benefits functions.

The cost and benefit functions are inherently non-linear in this design problem, with the discrete solution space. To reduce computational complexity, cost functions of the custom power devices are discretized, and the benefit functions are approximated using piecewise constant functions. In this regard, the objective of this problem is to maximize the aggregated utility generated by all the participants, subject to each participant strictly receiving more utility than what they can generate on their own. This requires the problem to be solved in two stages. It has been shown that the sought approximations have no impact on the outcome and can be applied to devices with a discrete cost function. The proposed problem formulation is demonstrated using three devices, three custom solutions, and three customer problem. Notably, the proposed methodology would generate more utility to the customers than the methods where the customers are expected to pay a premium in return for the improved PQ grade.

CRediT authorship contribution statement

Subir Majumder: Conceptualization, Methodology, Investigation, Visualization, Writing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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